Off-topic: a new interpretation of Weak Kleene logic

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This paper offers a new and very simple alternative to Bochvar's well-known *nonsense* – or *meaninglessness* – interpretation of Weak Kleene logic [3]. To help orient discussion I begin by reviewing the familiar Strong Kleene logic [8] and its standard interpretation; I then review Weak Kleene logic and the standard (viz., Bochvar) interpretation. While I note a common worry about the Bochvar interpretation my aim is only to give an alternative – and I think very elegant – interpretation, not necessarily a replacement. The new interpretation is given in §4, specifically in §4.3.

1 *Strong Kleene and its standard interpretation*

Strong Kleene logic (viz., K3) is familiar in philosophy.[1] The set $\{1, .5, 0\}$ of semantic values has a natural interpretation: the value 1 is read as *true*; the value 0 is read as *false*; and the third value 0.5 is read as *gappy* – neither true nor false.[2] This interpretation of the values is intended to motivate the K3 treatment of connectives reflected in the familiar tables:

[1]For both Strong and (the target) Weak Kleene logics see [8] and secondary discussion [2, 7, 10].

[2]Strictly, this is relative to a model (i.e., when the value of sentence φ has value 1 in a model, φ is said to be *true according to the model*) but I leave this as implicit throughout.

$\wedge$	1	.5	0
1	1	.5	0
.5	.5	.5	0
0	0	0	0

$\vee$	1	.5	0
1	1	1	1
.5	1	.5	.5
0	1	.5	0

$\neg$	
1	0
.5	.5
0	1

In particular, a conjunction is false if at least one conjunct is false, and a disjunction is true if at least one disjunct is true. Having a (meaningful) gappy sentence as a component doesn't take away from the truth of a disjunction that has at least one true disjunct, or from the falsity of a conjunction with at least one false conjunct. Meaningful-but-gappy sentences result in compound gaps only if there are insufficient classical-logic grounds for their truth or falsity.

So much for the standard *gappy* interpretation of K3, which I assume to be familiar. I turn to the target Weak Kleene logic.

2 *Weak Kleene and the Bochvar interpretation*

Strong Kleene logic has a 'weak' sibling, namely, Weak Kleene logic (WK3), which is the focus of this paper. WK3 uses the set $\{1, .5, 0\}$ of semantic values but has a strikingly different approach to connectives from K3:

$\wedge$	1	.5	0
1	1	.5	0
.5	.5	.5	.5
0	0	.5	0

$\vee$	1	.5	0
1	1	.5	1
.5	.5	.5	.5
0	1	.5	0

$\neg$	
1	0
.5	.5
0	1

Like K3 the treatment of connectives is exactly per classical logic when 0.5 is completely absent. (To see this, simply block out the third row of each table and third column of each binary-connective table.) Strikingly unlike K3 a disjunction fails to be true if at least one disjunct is true, and a conjunction fails to be false if at least one conjunct is false. What motivates this prima facie puzzling treatment of connectives?

Here is where a philosophical interpretation comes into play. And here is where Bochvar's famous 'meaningless' (or 'nonsense') interpretation [3] is standardly invoked. On this interpretation the values 1 and 0 are read exactly as in K3 (viz., true and false, respectively); however, the third value 0.5 is read as *meaningless*, so that φ's having value 0.5 is read as φ's being meaningless (in the given model). This interpretation is standardly taken to motivate the given treatment of the connectives: a conjunction is meaningless if at least one conjunct is meaningless; and the same with a disjunction; and the same with a negation.[3]

The WK3 consequence relation (the logic) is defined in terms of preserving the value 1 (i.e., being true). An important *invalidity* in WK3 is the failure of Addition (where addition is the form $\varphi \therefore \varphi \vee \psi$), namely:

$$\varphi \nvdash_{wk3} \varphi \vee \psi.$$

A counterexample is any model in which φ has value 1 and ψ has value 0.5, in which case (see table for disjunction) $\varphi \vee \psi$ has value 0.5.

One might find the failure of Addition to be strange, but the point of Bochvar's interpretation is to explain or motivate its failure. And the common view, which I do not aim to undermine, is that Addition ought to fail if a meaningless disjunct can sneak into the picture, since – at least on the driving background picture of 'nonsense' or 'meaninglessness' – the entire disjunction would thereby be meaningless. A meaningless bit makes the whole thing meaningless. That's Bochvar's idea.

3 *A worry about the Bochvar interpretation*

One common worry about Bochvar's interpretation is just this: when φ is genuinely meaningless there's a clear question of whether φ is even apt to be logically conjoined (or disjoined or negated) to make a conjunction (disjunction, negation). The logical connectives are generally thought to take meaningful sentences – indeed, on some accounts (of 'propositional connectives'), *propositions* – as input. This commonly held understanding of the inputs of standard connectives is in prima facie tension with the Bochvar interpretation, at least on a flat-footed understanding of it.

I share the given worry but am not endorsing it as an objection against the Bochvar interpretation of WK3. Instead, I propose another interpretation which is simple, elegant, and no less natural than the Bochvar interpretation.

[3] On this interpretation the third value is thought of as *infectious* and the treatment of connectives is sometimes summarized by the slogan 'a jot of rat's dung [i.e., this infectious value] spoils the soup' [2]. (The slogan's use in this context is often attributed to Bas van Fraassen.)

4 *A new interpretation*

4.1 *Terminology: theories*

Towards setting terminology think of a *theory* in the logician's sense: namely, a *closed theory*. In particular, think of a theory as a set of sentences closed under a consequence relation. Following Tarski we think of a theory as the set $Cn(T)$ of all consequences of T for some set of claims T, namely, $Cn(T) = \{\varphi : T \vdash \varphi\}$, where $\vdash$ is a consequence relation. So, for example, if one were to close a set X of sentences under K3 (or many other very familiar logics) then $\varphi \vee \psi$ is in the resulting theory (viz., $Cn(X)$) if φ is in the theory; however, since Addition fails in WK3, closing X under WK3 consequence won't put $\varphi \vee \psi$ in $Cn(X)$ simply because φ is in the theory. For present purposes the important (though terminological) point is that different consequence relations – different 'logics' – can result in different theories even if one starts with the same initial (pre-closed) set of truths.

4.2 *Motivating ideas*

The proposed interpretation is motivated by the following ideas, all of which I take to be prima facie plausible (and offer here without argument).

1. A theory is about all and only what its elements – that is, the claims in the theory – are about.
2. Conjunctions, disjunctions and negations are about exactly whatever their respective subsentences are about:
   (a) Conjunction $\varphi \wedge \psi$ is about exactly whatever φ and ψ are about.
   (b) Disjunction $\varphi \vee \psi$ is about exactly whatever φ and ψ are about.
   (c) Negation $\neg\varphi$ is about exactly whatever φ is about.
3. Theories in English are rarely about every topic expressible in English.

Putting these simple ideas together tells against using mere truth-preserving consequence as a relation under which all theories in English are closed. After all, Addition is truth-preserving; however, it isn't topic-preserving given (2b) and (3). As per (2b) the disjunction of *Grass is green* and *Brexit happened* is about grass and Brexit; but, by way of a witness for (3), our true theory of grass is not about Brexit. Addition can take theories off-topic.[4]

4.3 *The proposed interpretation: off-topic*

The foregoing ideas motivate a very simple but attractive interpretation of WK3 as a logic that concerns not simply truth-preservation but *truth-and-topic preservation*, where *being off-topic* is absolute.[5] The proposal: read the value 1 not simply as *true* but rather as *true and on-topic*, and similarly 0 as *false and on-topic*. Finally, read the third value 0.5 as *off-topic*. This interpretation, I claim, motivates the WK3 treatment of connectives, at least if the background motivating ideas (see §4.2) are held fixed.

Being on-topic, on this conception, is an absolute affair: φ is either true-and-on-topic, false-and-on-topic or φ is off-topic – full stop. And given this conception the WK3 treatment gets things right. Consider the on-topic cases: so long as all sentences are *on-topic* – that is, that no sentence is off-topic (i.e., no sentence has value 0.5) – then the connectives enjoy a perfectly classical-logic treatment. Consider the off-topic case. Let φ be off-topic. Then – given (2a) – the conjunction of φ and any sentence ψ is off-topic. The same goes for disjunction: a disjunction is off-topic if one of its disjuncts is off-topic. And similarly for negation: $\neg\varphi$ is off-topic given that φ is off-topic.

Not only does the proposed interpretation motivate the WK3 treatment of the connectives in a simple and natural way; it motivates the account of consequence as preserving value 1. In particular, while K3 cares only about truth-preservation the concern of WK3, on the current interpretation, is preservation of on-topic-truth (so to speak); and this motivates treating only value 1 – namely, *true-and-on-topic* – as 'designated' (i.e., the value to be preserved by the consequence relation).

[4] This observation about topics and Addition is not new; it is anticipated in work by Demolombe and Jones [5], and even by Goodman [6] and by Buvač et al [4], but none of these authors advance the proposed interpretation of WK3 I give here. In Demolombe & Jones' work, which is closest to my interest here, the proposed interpretation of WK3 is absent; and indeed Demolombe & Jones reject essential background parts of my interpretation such as (2b) and related principles. (The Demolombe & Jones work may be conflating *is relevant to topic t* and *is about topic t*, which might explain their rejection of target principles.) Pioneers of the *relevant-to* idea are Richard Angell [1] and William Parry [9] – work that continues to inspire further work in the relevance/relevant-logic tradition.

[5] An alternative account might explore 'partially off-topic', but I do not see this as delivering a natural interpretation of WK3 (which is my only concern here).

5 *Closing remark*

While the two interpretations are different I note that the new interpretation of WK3 need not be seen as a direct competitor to the Bochvar interpretation. Not only need there be no interesting competition among such interpretations (e.g., they can motivate different applications); one might even think of *being off-topic* as one way of being 'meaningless'. Maybe. My hope is only that this paper puts the 'off-topic' interpretation on the philosophical table.[6]

References

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[6] I am very grateful to Rohan French, Greg Restall and Steve Yablo for discussion of the proposed interpretation, and to the Editor (viz. Edwin Mares) whose comments greatly improved the paper.

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