On truth, abnormal worlds, and necessity

Jc Beall*

1 Introduction

Various semantic theories (e.g., truth, exemplification, and more) are underwritten by so-called depth-relevant logics. Such logics afford non-trivial theories that enjoy unrestricted semantic principles (e.g., T-biconditionals, comprehension, etc.). Standard semantics for such logics are so-called non-normal-worlds semantics, which add 'abnormal worlds' (or 'non-normal worlds') to an otherwise standard possible-worlds framework. (All of these ideas are briefly reviewed below.)

Once worlds (of any sort) are in the picture, questions about other worlds-involving notions emerge. One issue concerns the addition of standard alethic modalities—e.g., necessity—into the picture. In this paper, I note that the addition of such modalities (e.g., necessity, on which I focus here) is not entirely straightforward. In particular, the problems that motivate the target (depth-relevant) semantic theories—namely, Curry-paradoxical problems—equally constrain the treatment of alethic modalities.[1]

The paper runs as follows. §2–§3 review Curry's paradox and its upshot for target semantic theories. §4 sketches the target (abnormal-worlds) semantics as background to the main issue. The main issue is discussed in §5 and §6. A solution to the target problem is given in §7, with §8 giving a few closing remarks.

2 Robust contraction freedom

A lesson commonly drawn from Curry's paradox is that semantic principles (e.g., truth biconditionals, exemplification or comprehension biconditionals, etc.) need

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[1] I concentrate solely on necessity in this paper. A more general result concerning other modalities is discussed in a separate paper (Beall, 2009a).

to be free from various forms of contraction (Meyer, Routley, & Dunn, 1979; Priest, 2006b; Field, 2008; Beall, 2009b). A simple way of seeing the point is via a result of Greg Restall's (1993a).

Let $\rightarrow$ be a rule-detachable conditional (our target detachable arrow),[2] and let $\odot$ be a binary connective. Following Restall (1993a), we call $\odot$ a *contracting connective* if all of the following hold.[3]

$$A \rightarrow B \vdash A \odot B \tag{1}$$

$$A \odot (A \odot B) \vdash A \odot B \tag{2}$$

$$A,\ A \odot B \vdash B \tag{3}$$

For Curry-paradoxical reasons, any contracting connective trivializes a (sufficiently expressive) language that enjoys a truth predicate for which all of the target-arrow biconditionals hold (where $\leftrightarrow$ is formed via our target detachable arrow and conjunction in the usual way):[4]

$$Tr(\langle A \rangle) \leftrightarrow A \tag{4}$$

To see this, let $\odot$ be a contracting connective, and $\bot$ an explosive sentence (i.e., implies all sentences), and let k name the sentence $Tr(k) \odot \bot$, so that we have the following true identity statement:

$$k = \langle Tr(k) \odot \bot \rangle \tag{5}$$

The target (Curry-) instance of (4) is

$$Tr(\langle Tr(k) \odot \bot \rangle) \leftrightarrow Tr(k) \odot \bot \tag{6}$$

From this and our identity (5) we get

$$Tr(k) \leftrightarrow Tr(k) \odot \bot \tag{7}$$

Applying (1) to the LRD of (7) yields

$$Tr(k) \odot (Tr(k) \odot \bot) \tag{8}$$

[2]By *rule-detachable* is meant that, according to the given logic (or consequence relation $\vdash$), A and $A \rightarrow B$ jointly imply B, that is, that the argument from from A and $A \rightarrow B$ to B is valid (according to the given logic). Henceforth, I use 'detachable' just to mean *rule-detachable*.

I should also note, with respect to notation, that I use the turnstile (throughout) to record the validity relation; it may be read as *implies*, so that '$A \vdash B$' amounts to the claim that A implies B or—equivalently—that B is a consequence of A.

[3]Restall calls it a 'contracting implication', but I will avoid this terminology. (Some folks might worry about whether it's really an *implication* or even a *conditional* or etc., but this is irrelevant to the current discussion, and so I simply sidestep by using the generic 'connective'.)

[4]Think of $\langle A \rangle$ as a structural-descriptive name of A (or, if you wish, Gödel codes, or some such suitable naming device). Also, we assume that conjunction is standard (obeying both Adjunction and Simplification), that is, $A, B \vdash A \wedge B$ and $A \wedge B \vdash A$ (similarly for B).

Applying (2), the basic contraction rule, to (8) yields

$$Tr(k) \odot \perp \tag{9}$$

But, now, (9) and the RLD of (7) deliver

$$Tr(k) \tag{10}$$

The final blow comes from (3), which, applied to (9) and (10), delivers $\perp$, which implies all sentences. Triviality—everyone's worst nightmare.

3 The upshot

For convenience, let us call the conditional involved in one's semantic principles (e.g., truth biconditionals, exemplification or naïve-membership comprehension, and so on) a *semantic conditional.* Let us assume, as I will throughout, that our semantic conditional is a *detachable* conditional.[5]

An immediate upshot of §2 is that, on pain of its being a contracting connective, one's semantic conditional—say, the conditional in the truth biconditionals—cannot satisfy (2). (Letting $\odot$ be $\rightarrow$ in (1) and (3) makes the point plain.) The more general point is that having a non-contracting semantic conditional is not itself sufficient to avoid Curry problems. What one cannot have is *any* contracting connective in the language, lest the resulting semantic theory be trivial (via the considerations in §2). Following Restall (1993a), we say that a language (or theory in the given language) is *robustly contraction-free* just if it is free of a contracting connective.

4 Abnormal worlds and depth-relevant theories

There are a variety of logics that provide robustly contraction-free semantic theories. My concern here is a family of so-called depth-relevant logics, which have been used to provide non-trivial semantic theories (Brady, 1989; Priest, 2006a, 2006b; Beall, 2009b).[6]

The target logics enjoy a possible-worlds semantics called *abnormal-worlds semantics* or, more commonly, *non-normal-worlds semantics.*[7] The point of this section is not to sketch the full semantic framework(s) in question, but rather to sketch just enough of the target framework to raise the main issue of the paper (concerning the addition of necessity into the mix).

[5]I myself endorse such a position in (2009b), as do Priest (2006b, 2006a), Field (2008), and many other theorists. I should note, however, that I have lately come to seriously question this assumption, but discussion of that topic is for another work (Beall, 20++).

[6]Though not depth-relevant, the recent theories of Field (2008) and Brady (2006) fall into the same target family of semantics theories; however, there are serious differences from the target depth-relevant theories, and I do not discuss the Brady/Field (or their ilk) theories further here.

[7]Kripke (1965) first invoked non-normal worlds to model weak C. I. Lewis modal systems, but they're now useful for other things—epistemic logics, and, on topic here, for conditionals.

** *Parenthetical remark.* I should note that my real target semantics are so-called simplified Routley–Meyer semantics that involve a ternary relation on worlds (Routley & Meyer, 1973). It is this ternary relation that is invoked to achieve a detachable but contraction-free semantic conditional in some of the target semantic theories (Priest, 2006b; Beall, 2009b), and in many ways is at the heart of (at least worlds) semantics for relevant logics (Priest & Sylvan, 1992; Restall, 1993b). But simply for simplicity here, I present a different, 'arbitrary-evaluator' framework (Routley & Loparić, 1978; Priest, 1992; Beall, 2005). *End parenthetical.* **

4.1 Abnormal-worlds structures

Although many semantic paradoxes arise only at the predicate–quantifier level, the main ideas of this paper can be conveyed at the propositional level. So, for simplicity, we concentrate only on a simple (positive) propositional language with $\wedge$, $\vee$, and $\rightarrow$. (As mentioned above, we can ignore negation. In fact, we can even ignore conjunction and disjunction for present purposes, but I sketch their treatment for purposes of comparison with the 'jumpy' treatment of the arrow.)

Our (abnormal-worlds) interpretations are much like standard possible-worlds interpretations except for an additional non-empty set $\mathcal{N}$ comprising the *normal worlds* of the interpretations. Abnormal-worlds structures are pairs $\langle \mathcal{W}, \mathcal{N} \rangle$, where $\mathcal{W}$ is a non-empty set of worlds (more neutrally, points) and $\mathcal{N}$ a non-empty subset of $\mathcal{W}$. If $x \in \mathcal{N}$ we call x a *normal world* (or normal point), and if $x \in \mathcal{W} \setminus \mathcal{N}$ we call x an *abnormal* world. (NB: $\mathcal{W} \setminus \mathcal{N}$ may be empty.)

We let $\models$ be a truth-at-a-point relation, relating sentences to worlds. In particular, $\models$ may be any subset of $\mathcal{W} \times \mathcal{L}$ (where $\mathcal{L}$ comprises our sentences). If $x \models A$ we say that A is (at least) true at x.[8]

For present purposes, we say that any abnormal-worlds structure, combined with a truth-at-a-point relation, is an *abnormal-worlds interpretation.* (We specify 'admissible interpretations' or models below.)

4.2 Models: truth conditions

A characteristic feature of abnormal-worlds semantics is that the truth conditions for connectives may be 'jumpy' or 'non-uniform' across types of points: a connective might behave differently—have different truth-at-a-point conditions—at different types of points. For convenient terminology, we say that a connective is *jumpy* or *non-uniform* iff its truth-at-a-point conditions vary across normal and abnormal worlds; otherwise, we call the connective *uniform.* (The distinction will be clear from the truth conditions for the conditional below.)

Models. In the target semantics, conjunction and disjunction are uniform, but the target 'semantic conditional' is jumpy. In particular, we call any abnormal-worlds interpretation a *model*—or, if you like, *admissible interpretation*—if and

[8] The parenthetical 'at least' flags that the target theories are paraconsistent, allowing sentences to be true and also false at a point. For present purposes, however, we can ignore this issue—focusing only on a *truth-at-a-world* relation (ignoring falsity-at-a-point).

only if it 'obeys' the following 'truth conditions' for our connectives. (For present purposes, we can ignore 'falsity conditions' for our connectives.)

1. Conjunction:
   - Normal or Abnormal: for any $x \in \mathcal{W}$
   $$x \models A \wedge B \text{ iff } x \models A \text{ and } x \models B$$
2. Disjunction:
   - Normal or Abnormal: for any $x \in \mathcal{W}$
   $$x \models A \vee B \text{ iff } x \models A \text{ or } x \models B$$
3. Conditional:
   - Normal: for any $x \in \mathcal{N}$
   $$x \models A \rightarrow B \text{ iff } y \not\models A \text{ or } y \models B \text{ for } \textit{any } y \in \mathcal{W}$$
   - Abnormal: for any $x \in \mathcal{W} \setminus \mathcal{N}$
   $$x \models A \rightarrow B \text{ iff } \ldots \text{let this be arbitrary!}$$

The foregoing truth conditions are familiar except, perhaps, for the conditional's truth-at-a-point conditions at *abnormal worlds*. At abnormal worlds, the truth (or semantic status, generally) of $A \rightarrow B$ is entirely arbitrary.[9]

Validity. We define *validity* only over normal worlds (of all models): the argument from $A_1, \ldots, A_n$ to B is valid iff there's no *normal* world of any model at which each A_i is true but B not.

4.3 Example

Note that the foregoing framework delivers a detachable but non-contracting conditional. Letting $\vdash$ be our validity relation, we have detachment:

$$A, A \rightarrow B \vdash B$$

Suppose, for reductio, that there's a point $x \in \mathcal{N}$ at which A and $A \rightarrow B$ are true but B untrue (i.e., $x \not\models B$). Since, by supposition, $x \models A \rightarrow B$ and $x \in \mathcal{N}$, there's no point $y \in \mathcal{W}$ such that $y \models A$ and $y \not\models B$, and a fortiori either $x \not\models A$ or $x \models B$. Contradiction.

With respect to contraction, we have

$$A \rightarrow (A \rightarrow B) \nvdash A \rightarrow B$$

[9] I should repeat that, in the target logics, one need not make the conditional's status arbitrary at abnormal worlds, but this approach simplifies matters for current discussion. For discussion of this 'arbitrary' approach, see Priest, 1992 and Beall, 2005.

In particular, let $\mathcal{W} = \{x, y\}$ with $\mathcal{N} = \{x\}$. Let $x \not\models A$. In turn, let $y \models A$ and $y \not\models B$ but $y \models A \rightarrow B$. In this model, there's no point in which A is true but $A \rightarrow B$ untrue, and hence, since x is normal, we have $x \models A \rightarrow (A \rightarrow B)$. On the other hand, there is a point at which A is true but B untrue (viz., y, which is abnormal); and so $x \not\models A \rightarrow B$.

5 Target issue: necessity

While there are a host of philosophical questions that surround the given abnormal-worlds semantics, my interest here is in a relatively unexplored one: namely, the addition of necessity into the mix. In what follows, I note that, on pain of generating a contracting connective (and, hence, trivializing the target theories), a necessity operator along standard *quantifier-over-all-worlds* lines must be 'jumpy' or non-uniform.

5.1 Uniform, all worlds

Let us assume that we've added a unary connective $\Box$ to be our (broad) necessity operator (viz., *it is necessary that...*). One natural thought for truth conditions is a uniform, all-worlds condition:

- Normal or Abnormal: for any $x \in \mathcal{W}$

$$x \models \Box A \text{ iff } y \models A \text{ for all } y \in \mathcal{W}$$

This is the standard idea prevalent in philosophy: namely, that our broad-necessity operator ranges over *all* worlds, and (for 'uniformity') does as much at *all* points. (If we think in terms of an 'access relation' on worlds, then the idea is that we have an equivalence relation on worlds.) The problem with this is that, not surprisingly (Kripke, 1965), we can now (i.e., in our *abnormal*-worlds framework) have logical truths that do not count as necessarily true. For example, one may easily check that $A \rightarrow A$ is logically true (i.e., no countermodel), but there are many worlds (of many models) at which $A \rightarrow A$ is untrue—namely, abnormal worlds. At least on the surface, this break between alethic necessity and logical truth seems awkward. On the other hand, perhaps the given break between aletheic necessity and logical truth—more formally, a failure of Necessitation—is not only what one should expect in the current (abnormal-worlds) conext; it's what one should want, since abnormal-worlds are strange (e.g., $A \rightarrow A$ can fail to be true!), so strange that we define validity only over the normal worlds. I will not argue the matter here.

We can allow (if we want) that our broad notion of necessity ought to be as broad as the universe of points that we recognize, and that if (as we're supposing) we recognize abnormal points, then our broad-necessity operator ought to range (i.e., quantify over) them too. Still, one might (quite reasonably) think that we have another notion of necessity, namely, one that quantifies only over *normal* worlds—the 'real possibilities', so to speak. This is the notion of necessity at issue in this paper.

5.2 Uniform, all *normal* worlds

While abnormal worlds might be strange enough to be 'possibilities' only in some very charitable sense, it is natural to take the *normal worlds* to be the 'real possibilities' involved in our alethic-necessity (or, dually, possibility) claims.

- Normal or Abnormal: for any $x \in \mathcal{W}$

$$x \models \Box A \ \text{ iff } \ y \models A \text{ for all } y \in \mathcal{N}$$

In short: we take our necessity claims to quantify over all and only the *normal* worlds. Not only does this deliver the necessity of all logical truths; it also accommodates the general intuition that the abnormal worlds are 'impossible worlds' of some sort (Priest, 1992; Caret, 2009), or at any rate that abnormal worlds are beyond the intended range of our necessity claims.

It is not difficult to see that we also get a lot of standard logical behavior for the Box. One notable feature is the following lemma (not unfamiliar from the more standard S5 setting).

- *Box Lemma.* For any model, $\Box A$ is true at all worlds or true at none.

  Proof. This is fairly clear from the truth conditions for the box. In short: either A is true at all *normal* worlds or not. If the former, $\Box A$ is true everywhere (by the given truth conditions). If the latter, $\Box A$ is true nowhere (by the given truth conditions).

Of course, whether one gets standard S5 interaction between the Box and Diamond depends on how negation is treated, at least if we're defining the Diamond in terms of negation and the Box along standard lines. But this issue is beyond the limited aims of this paper.

6 Trouble: contracting connective

However natural the uniform-all-normal-worlds account of §5.2 may be, it cannot be utilized in the target semantic theories. The trouble, in short, is that the account breeds a contracting connective.

To see the result, let us define a connective $\Rightarrow$ as follows:

$$A \Rightarrow B \ := \ \Box(A \rightarrow B)$$

Our defined connective is a contracting connective. To see this, first note a general lemma concerning the arrow.

- *Arrow Lemma.* For any model, $A \rightarrow B$ is true at some normal world if and only if it is true at all normal worlds.

Proof. This is fairly clear from the truth-at-a-*normal*-point conditions for the arrow: $A \to B$ is true at a normal world if and only if there's *no point whatsover* at which A is true and B untrue. So, $A \to B$ is true at *some* normal point iff true at *all* normal points. (Think about the Box in an S5 setting. The situation is the same here when we restrict to normal worlds.)

The conditions for contracting connectives are now met as follows.

1. $A \to B \vdash A \Rightarrow B$.

   Proof. This follows from the Arrow Lemma and the truth conditions for the Box. A countermodel would require a normal world at which $A \to B$ is true but also (to make the conclusion untrue) a *normal* world at which $A \to B$ is untrue. This contradicts the Arrow Lemma.

2. $A \Rightarrow (A \Rightarrow B) \vdash A \Rightarrow B$.

   Proof. Suppose that $x \in \mathcal{N}$ and $x \models \Box(A \to \Box(A \to B))$, in which case, via the Box's truth conditions, $x \models A \to \Box(A \to B)$. Suppose, for reductio, that $x \not\models \Box(A \to B)$, in which case there's some $y \in \mathcal{N}$ such that $y \not\models A \to B$, and so—via the Arrow's normal-point conditions—some $z \in \mathcal{W}$ such that $z \models A$ and $z \not\models B$. Now, given that $x \not\models \Box(A \to B)$, the Box Lemma implies that $z \not\models \Box(A \to B)$. But, then, there's a point (viz., z) at which A is true but $\Box(A \to B)$ untrue—a point at which A is true but $A \Rightarrow B$ untrue. Hence, by the arrow's normal-point conditions, $x \not\models A \to (A \Rightarrow B)$. Contradiction (see initial supposition).

3. $A, A \Rightarrow B \vdash B$.

   Proof. Let $x \in \mathcal{N}$ and $x \models A$ and $x \models A \Rightarrow B$. For reductio, suppose that $x \not\models B$, in which case $x \not\models A \to B$ since there's a point (viz., x) at which A is true but B untrue. But this contradicts the supposition that $x \models A \Rightarrow B$, which requires that $A \to B$ be true at all normal worlds—and, a fortiori, true at x.

The upshot: we cannot recognize an alethic-necessity operator that *uniformly* ranges over only normal worlds. What to do?

7 Solution: jumpy necessity

The solution—at least if (as I'm supposing) we want Necessitation to hold for our target necessity operator—is to give up on uniformity and treat our Box like we treat our arrow: namely, as a jumpy or non-uniform connective. And this makes sense. After all, we first invoked abnormal worlds to free our conditional from contraction—freedom from feature (2) and its ilk. The conditional (i.e., our basic arrow) avoids contraction by going on holiday at abnormal worlds: it behaves differently—indeed, on the simple sketch here, arbitrarily—at abnormal worlds. (Look again at the countermodel to contraction in §4.3.) The trouble with our uniform, all-normal-worlds approach to the Box is that, because it is uniform, the arrow, when 'boxed up' (so to speak), is forced to behave normally; it is not allowed to be evaluated in its holiday state. In particular, while $A \rightarrow B$ can go on holiday at abnormal worlds, $\Box(A \rightarrow B)$ forces $A \rightarrow B$ to return (so to speak) to normal worlds for evaluation, and so our defined arrow $A \Rightarrow B$ never has a point at which to shake off contraction.

We can skip further metaphor and simply go to a solution. As above, the solution is to treat our Box as we treat the arrow (our other 'intensional' connective): treat it as a jumpy connective. There are many ways to do this, but the simplest (though, perhaps, philosophically ugliest) is to again invoke an 'arbitrary evaluator' at abnormal points. And to retain Necessitation (i.e., that if A is logically true, so too is $\Box A$), we retain the spirit of our all-normal-worlds condition at (and only at) normal worlds.

- Normal: for $x \in \mathcal{N}$

$$x \models \Box A \text{ iff } y \models A \text{ for all } y \in \mathcal{N}$$

- Abnormal: for $x \in \mathcal{W} \setminus \mathcal{N}$

$$x \models \Box A \text{ iff } \ldots \text{let this be arbitrary!}$$

With this approach, we keep Necessitation.

- *Necessitation.* If $\vdash A$ then $\vdash \Box A$.

  Proof. This follows immediately from the fact that validity (even for sentences) is defined only over normal worlds: $\vdash A$ iff $x \models A$ for all $x \in \mathcal{N}$ in all models.

Moreover, and for present purposes more importantly, we avoid $A \Rightarrow B$'s being a contracting connective—where, again, $A \Rightarrow B$ is $\Box(A \rightarrow B)$.

- $A \Rightarrow (A \Rightarrow B) \nvdash A \Rightarrow B$.

  Countermodel. The same countermodel to (2) works here. Let $\mathcal{W} = \{x, y\}$ with $\mathcal{N} = \{x\}$. Let $x \not\models A$. Now, let $y \models A$ and $y \not\models B$ but $y \models \Box(A \rightarrow B)$. [Recall that box claims can be whatever we want at abnormal points.] In this model, there's no point at which A is true but $A \Rightarrow B$ untrue, and hence, since x is normal, we have $x \models A \rightarrow (A \Rightarrow B)$. Hence, since x is the sole normal world, we have $x \models \Box(A \rightarrow (A \Rightarrow B))$, that is, $x \models A \Rightarrow (A \Rightarrow B)$. On the other hand, there is a point (viz., y) at which A is true but B untrue; and so, by the Arrow's normal-point conditions, $x \not\models A \rightarrow B$. By the Box's normal-point conditions, we have that $x \not\models \Box(A \rightarrow B)$, that is, $x \not\models A \Rightarrow B$.

While there may be other features of necessity that we may want, this *non-uniform* or 'jumpy', all-normal-worlds approach is at least promising.

*** Parenthetical remark.* Showing that $\Box(A \rightarrow B)$ fails to contract is insufficient for establishing robust contraction freedom, but for semantic theories in the ballpark (Priest, 2006b; Beall, 2009b), a non-triviality result is available. For example, such theories have one-normal-world models (with all other worlds abnormal). So, as Greg Restall (in conversation) noted, $\Box A$ is vacuously true in such models (i.e., true at the unique normal world of such models), and so we have models of the resulting Box-ful semantic theories (with the Box having our given 'jumpy' all-normal-worlds semantics). Of course, what one would like are *natural* models of such theories, where not *all* Box claims are true, and this—at the time of writing—remains an open problem. [The target sense of 'natural' is imprecise and relative to background philosophical issues concerning the target semantic notions, but an example of a *natural model* for a Box-free semantic theory, relative to a particular transparent conception of truth, is presented in my *Spandrels of Truth* (2009b), with the model essentially due to Ross Brady, Chris Mortensen (1995), and Graham Priest.] *End parenthetical. ***

8 Concluding remarks

Contraction-free logics remain popular (and, I think, promising) routes towards constructing rich semantic theories. Among such logics are depth-relevant logics, which enjoy a familiar (though abnormal-) worlds semantics. Little attention has been put to the issue of adding other philosophically important notions into the mix, probably because the addition of such operators seemed on the surface to be straightforward. (This is certainly why I gave the matter little thought until recently.) It is surprising that there is any issue at all. What this paper shows is that, while the solution (e.g., §7) is straightforward, care must nonetheless be taken when adding otherwise very familiar (and philosophically important) intensional notions into the mix.

*** Parenthetical remark.* There is a more general result concerning 'talking about normal worlds' from which the present result about necessity (or other modal notions like Actuality, etc.) follows. I hope to publish the broader result, with more general discussion, in a larger paper (Beall, 2009a). *End parenthetical. ***

Jc Beall
Philosophy Department and UConn Logic Group
University of Connecticut
Storrs, CT 06269–2054 USA
`jc.beall@uconn.edu`
`homepages.uconn.edu/~jcb02005`
`logic.uconn.edu`

References

Beall, J. (20++). *Transparent truth, limited inconsistency, and detachment.* (To appear.)

Beall, J. (2005). Transparent disquotationalism. In J. Beall & B. Armour-Garb (Eds.), *Deflationism and paradox* (pp. 7–22). Oxford: Oxford University Press.

Beall, J. (2009a). *Contraction and normal worlds: an overspill result.* (To appear. Presented at the 2009 AAL in Melbourne, Victoria University in Wellington, and University of Otago in Dunedin.)

Beall, J. (2009b). *Spandrels of Truth.* Oxford: Oxford University Press.

Beall, J., Brady, R., Hazen, A., Priest, G., & Restall, G. (2006). Relevant restricted quantification. *Journal of Philosophical Logic*, *35*(6), 587–598.

Brady, R. T. (1989). The non-triviality of dialectical set theory. In G. Priest, R. Routley, & J. Norman (Eds.), *Paraconsistent logic: Essays on the inconsistent* (pp. 437–470). Philosophia Verlag.

Brady, R. T. (2006). *Universal logic.* Stanford: CSLI Publications.

Caret, C. (2009). *Non-normal, but not impossible.* (To appear. Available at `http://sites.google.com/site/colincaret/workinprogress`)

Field, H. (2008). *Saving Truth from Paradox.* Oxford: Oxford University Press.

Hájek, P., Paris, J., & Shepherdson, J. (2000). The liar paradox and fuzzy logic. *Journal of Symbolic Logic*, *65*, 339–346.

Kripke, S. (1965). Semantical analysis of modal logic II: Non-normal modal propositional calculi. In J. W. Addison, L. Henkin, & A. Tarski (Eds.), *The Theory of Models* (pp. 206–220). Amsterdam: North-Holland Publishing Co.

Meyer, R. K., Routley, R., & Dunn, J. M. (1979). Curry's paradox. *Analysis*, *39*, 124–128.

Mortensen, C. (1995). *Inconsistent mathematics.* Kluwer Academic Publishers.

Priest, G. (1992). What is a non-normal world? *Logique et Analyse*, *35*, 291–302.

Priest, G. (2006a). *Doubt Truth To Be A Liar.* Oxford: Oxford University Press.

Priest, G. (2006b). *In Contradiction* (Second ed.). Oxford: Oxford University Press.

Priest, G., & Sylvan, R. (1992). Simplified semantics for basic relevant logics. *Journal of Philosophical Logic*, *21*, 217–232.

Restall, G. (1992). Arithmetic and truth in Łukasiewicz's infinitely valued logic. *Logique et Analyse*, *139–140*, 303–312.

Restall, G. (1993a). How to be *Really* contraction-free. *Studia Logica*, *52*, 381–391.

Restall, G. (1993b). Simplified semantics for relevant logics (and some of their rivals). *Journal of Philosophical Logic*, *22*, 481–511.

Routley, R., & Loparić, A. (1978). Semantical analyses of Arruda-da Costa P-systems and adjacent non-replacement systems. *Studia Logica*, *37*, 301–320.

Routley, R., & Meyer, R. K. (1973). Semantics of entailment. In H. Leblanc (Ed.), *Truth, Syntax, and Modality* (pp. 194–243). North Holland. (Proceedings of the Temple University Conference on Alternative Semantics)