From Full Blooded Platonism
to
Really Full Blooded Platonism

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1. Full-Blooded Platonism

Let a *platonic entity* be an entity entirely free of causal powers. Let *platonism* be the view that at least some platonic entities exist. Restricted to mathematical entities, platonism is the view that at least some mathematical entities are platonic entites. Benacerraf's [1973] epistemic challenge to platonism is the challenge of explaining how we could gain knowledge of platonic entities.

In his [1998] Mark Balaguer argues that *full blooded platonism* is the *only* theory that can solve Benacerraf's epistemic challenge to platonism. The idea of full blooded platonism is this: Any mathematical entity that *can* exist *does* exist. More accurately:

(FBP) Every *consistent* mathematical theory truly describes some part of the mathematical realm.

How does FBP solve the epistemic challenge? The idea, crudely put, is that we need merely think about entities corresponding to consistent mathematical theories. Since all such theories are true, all such thoughts are true of some portion of platonic reality. But, then, the epistemic challenge is met.

Balaguer's strategy, then, is to increase ontology to the limit of consistent mathematics. By letting *every* consistent mathematical theory bloom one thereby solves Benacerraf's epistemic challenge. This, at any rate, is the strategy.

Whether Balaguer's strategy is ultimately successful is not an issue I wish to take up; that issue has been discussed sufficiently by others.[1] Instead, I wish only to note that if Balaguer's strategy does work, then his FBP is *not* the only theory that can meet Benacerraf's challenge, and moreover that it is not the most attractive contender.

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[1] See, in particular, both Cheyne [1999] and, especially, Colyvan and Zalta [1999].

PHILOSOPHIA MATHEMATICA (3) Vol. 7 (1999), pp. 322–325.

2. Really Full-Blooded Platonism

Consider, in particular, the following competitor, namely, *Really Full Blooded Platonism*:

(RFBP) Every mathematical theory—consistent and *inconsistent* alike—truly describes some part of the mathematical realm.

The difference between FBP and RFBP is that the latter but not the former admits *inconsistent theories*—theories explored at length, for example, in C. Mortensen [1995]. Such theories are (classically) inconsistent but nonetheless nontrivial. That is, such theories contain both A and $\sim A$, for some A in the language; this is the inconsistency side of such theories. Despite this, such theories are not trivial; that is, there is at least one B in the language such that B is not an element of the theory. All of this is captured nicely by noting that such theories are underwritten by so-called *paraconsistent logics*. A logic, $\mathcal{L}$, is paraconsistent iff its consequence relation, $\vdash$, is such that $A, \sim A \nvdash B$, for some A and B.[2] The very existence of inconsistent mathematics raises two important points with respect to Balaguer's project. I discuss each point briefly in turn.

The first point is this: Balaguer is wrong to claim that FBP is the *only* theory to solve Benacerraf's challenge. After all, FBP is supposed to solve the problem by expanding platonic heaven to such a degree that one's cognitive faculties can't miss it (as it were). (If you're having trouble hitting the target, then just make your target bigger! This is the heart of Balaguer's strategy.) But, then, since RFBP simply expands the heavens even further, then RFBP solves the problem if FBP does. Hence, Balaguer is wrong to point to FBP as a lone contender.

Before turning to the second point let me quickly set aside a natural worry. One might argue that expansion of platonic heaven is generally good, but expansion can also be bad. The objection is that RFBP leads quickly to the worst sort of expansion, namely, *trivialism*—expansion to the utmost trivial limit. The argument is this. If RFBP is true, then at least some contradictions—some instances of $A \wedge \sim A$—are true. But if some contradictions are true, then *everything* is true, which is easily shown via C. I. Lewis's familiar *independence argument*. Thus, if RFBP is admitted, then platonic heaven is indeed big, but it's too big—it bursts into triviality.

It is pretty clear, I think, that this sort of worry is a bad one on many fronts. The most critical problem, however, is that the Lewis independence argument is not valid in paraconsistent logics. This should be clear. After all, if, as inconsistent mathematics allows, $A \wedge \sim A$ really is true, then unless everything is true—which the classical logician denies—the argument from $A \wedge \sim A$ is an argument from truth to at least one untruth, B. Hence, the

[2] For an introduction to the rich field of paraconsistency see Priest and Sylvan [1989a], [1989b], and also Restall [1994]. See Mortensen [1995] for particular application to mathematics.

argument is not valid. For this reason, the objection doesn't get off the ground.

Consider, then, the second important point arising from the existence of inconsistent mathematics. We have seen above that FBP is *not* the only theory to solve Benacerraf's problem; RFBP does the trick just as well. But now notice that we have a decision to make between these rivals. Should we go with FBP or RFBP? The second point is that RFBP seems to be preferable.

Why is RFBP preferable? The reasoning can be put concisely. In short, FBP is either informal or not. If informal, then FBP is inconsistent; if not, then FBP is incomplete. If the former, then we should go with RFBP, since in effect we are there already. If the latter, then we should go with RFBP, since inconsistent mathematics affords completeness, which I take to be an obvious virtue. Either way, then, RFBP is preferable.

That formal theories buy their consistency at the price of completeness is well known. I will say a word, however, as to why *informal* mathematics is inconsistent. This can be seen by considering "Gödel's paradox', as Priest ([1987], p. 59) calls it. Consider the sentence:

(γ) γ is (informally) unprovable.

If γ is false, then it is provably true, and so true. By *reductio* γ is true. Since we have just (informally) proved γ, γ is (informally) provable. But since true, γ is (informally) unprovable. Contradiction.[3]

There seems to be little hope of denying that γ is indeed a sentence of our informal mathematics. Accordingly, the only way to avoid the above result is to revert to formalising away the inconsistency —a response familiar from the histories of naïve set theory, naïve semantic theory, and so on. If one does this, however, then (by familiar results) one loses *completeness*, which can be regained only by endorsing inconsistency.[4] Either way, then, we seem to be led to inconsistent mathematics.

3. Final Remarks

Balaguer argues for FBP, and argues that FBP alone can solve Benacerraf's epistemic challenge. I have argued that if FBP really can solve Benacerraf's epistemic challenge, then FBP is not alone in its capacity so to solve; RFBP

[3] Some might worry that paraconsistent logics (usually) reject *contraction*: $(A \to (A \to B)) \vdash A \to B$. The worry is that rejecting contraction leads naturally to rejecting *reductio*, which itself is used in the above proof that informal maths is inconsistent! The reply to this is that while contraction should indeed be rejected one can nonetheless retain *reductio*—and all classically logical truths, for that matter. For details on this see Priest's logic **LP**, which is nicely discussed in Priest [1987]. For other such standard results see Priest and Sylvan [1989b], and also Restall [1994], [to appear].

[4] That completeness can be captured in standard inconsistent theories is indicated in both Priest [1987] and Mortensen [1995].

can do the trick just as well. Moreover, there seems to be positive reason for endorsing RFBP. In short, FBP is either inconsistent or incomplete. If the former, we should endorse RFBP, since in effect we are there already. If the latter, we should endorse RFBP, since RFBP can give us the desired completeness.

Balaguer's strategy is ingenious in many ways. But if we really are going to expand platonic heaven in an effort to ensure our epistemic footing, then we need to explore the option of expanding heaven to its nontrivial limits. If this option is to be rejected, then we need good reason for rejecting it. For now, no such reason seems to exist.[5]

References

BALAGUER, MARK [1998]: *Platonism and Anti-Platonism in Mathematics*. Oxford: Oxford University Press.

BENACERRAF, PAUL [1973]: 'Mathematical truth', *Journal of Philosophy* **70**, 661–679.

CHEYNE, COLIN [1999]: 'Problems with profligate platonism', *Philosophia Mathematica* (3) **7**, 164–177.

COLYVAN, MARK, and EDWARD ZALTA [1999]: Review of Balaguer [1998]. *Philosophia Mathematica* (3) **7**, 336–349.

MORTENSEN, CHRIS [1995]: *Inconsistent Mathematics*. Dordrecht: Kluwer.

PRIEST, GRAHAM [1987]: *In Contradiction: A Study of the Transconsistent*. The Hague: Martinus Nijhoff.

PRIEST, GRAHAM, and RICHARD SYLVAN [1989a]: 'Introduction', in Priest, Sylvan, and Norman [1989], pp. xix–xxi.

_______ [1989b]: 'Systems of paraconsistent logic', in Priest, Sylvan, and Norman [1989], pp. 151-186.

PRIEST, GRAHAM, RICHARD SYLVAN, and JEAN NORMAN, eds. [1989]: *Paraconsistent Logic: Essays on the Inconsistent*. München, Hamden, Wien: Philosophia Verlag.

RESTALL, GREG [1994]: *On Logics without Contraction*. PhD Thesis. Brisbane: The University of Queensland.

_______ [to appear]: *An Introduction to Substructural Logics*. London: Routledge.

ABSTRACT. Mark Balaguer argues for *full blooded platonism* (FBP), and argues that FBP alone can solve Benacerraf's familiar epistemic challenge. I note that if FBP really can solve Benacerraf's epistemic challenge, then FBP is not alone in its capacity so to solve; RFBP—*really full blooded platonism*—can do the trick just as well, where RFBP differs from FBP by allowing entities from inconsistent mathematics. I also argue briefly that there is positive reason for endorsing RFBP.

[5] I am grateful to Mark Colyvan for discussion of Balaguer's position. Thanks to Chris Mortensen for discussion which improved this note significantly. Likewise, thanks to Graham Priest and Greg Restall for ongoing discussion of inconsistent theories.