Shrieking against gluts: the solution to the *'just true'* problem*

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1 *True and false* versus *just true (just false)*

Glut theorists say that some sentences (e.g., liar paradox) are true and false: both they and their negation are true. A common worry about glut theory, advanced under a variety of names, is the problem of 'just true' or 'consistent truth' or 'just false' or 'really not true' [1, 6, 7, 9, 10, 11, 14]. In a nutshell: glut theorists, by asserting $\alpha \wedge \neg\alpha$, can easily express that a sentence α is a glut; but a prima facie worry hovers over 'normal categories' involving, for example, claims that a sentence (or, generally, a theory) is *just true* (or, similarly, just false, etc.). As Graham Priest puts it,

> The challenge is a quite general one, but it assumes a particular significance in the context of the semantic paradoxes.... [It might be thought that if glut theorists] could express the thought that something is not ('*really* is not') true, unacceptable consequences might be expected to follow.... [12, p. 291]

And Priest, one of glut theory's main advocates, has said much on the matter. Indeed, by my lights, Priest [12, §20.4] advances the two main ingredients of the solution to the *just-true* problem: the 'shrieking idea', first advanced by Priest [12] and recently refined by Beall [3], and the observation that classical

*Forthcoming *Analysis*. This is not the final version.

theorists can claim no more than glut theorists – they rule out inconsistency *only up to triviality* (a matter explained below).

Unfortunately, there is a problem with Priest's actual proposal, which may be responsible for the ongoing worries about the 'just true' problem. In particular, while nonetheless going in the right direction, Priest's discussion of 'just true' is framed in a way that at least disguises the simplicity and naturalness of the core solution; and, indeed, the reply is incorrect as it stands. In this paper, I present the solution in its simple (and correct) form.

The structure of the discussion is as follows. In §2 I present Priest's target reply and discuss its main defect. §3, in turn, presents the core idea in terms of 'shriek rules', while §4 advances the main proposal.

2 *Incompatibility, consistency and collapse*

A central idea for all glut theorists is that *incompatibility* be seen disjunctively: α and β are incompatible just if their joint truth delivers triviality. This is 'disjunctive' in the sense that our theories containing both α and β are either untrue or trivial – that is, they're untrue *or* contain all sentences of the language of the theory. That incompatibility be understood as such serves as common ground for glut theorists and classical (or, generally, explosive-logic) theorists. The difference between such theorists shows up, if anywhere, in the would-be 'deliverer of triviality'. Let me explain.

The classical theorist (indeed, any explosive-logic theorist) finds incompatibility delivered by elementary *logic*: α and $\neg\alpha$ are logically incompatible; they're an 'explosive' mix, jointly implying (logically implying) triviality. But glut theorists, at least at the level of standard (boolean or first-order) vocabulary, reject that logic imposes any such explosive relationship [5]. If logic is to ensure incompatibility, it needs to come from somewhere other than the standard vocabulary.

Such a route is exactly what Priest [12] has long taken: he posits, in the tradition of relevance logicians, a logical connective in the language – indeed, as he sees it, a logical-entailment connective, true in virtue of what follows *logically* from what. And herein is where Priest delivers the core reply to the target just-true (or just-false, etc) problem:

> A dialetheist [glut theorist] can express the claim that something, α, is not true – in those very words, $\neg T\langle\alpha\rangle$. What she cannot do is ensure that the words she utters behave consistently: even

> if $\neg T\langle\alpha\rangle$ holds, $\alpha \wedge \neg T\langle\alpha\rangle$ may yet hold. But in fact, a classical logician [or any explosive-logic theorist] can do no better. He can endorse $\neg T\langle\alpha\rangle$, but this does not prevent his endorsing α as well.... [C]lassical logic, as such, is no guard against this. ...[A]ll the classical logician can do by way of saying something to indicate that α is not to be accepted is to assert something that will collapse things into triviality if he does accept α. But the dialetheist can do this too. She can assert $\alpha \rightarrow \perp$ [where the arrow here is Priest's posited 'entailment connective' in the tradition of relevance logic, and $\perp$ some 'explosive sentence' that implies triviality (e.g., 'All sentences are true')]. [12, p. 291]

The classical-logic theorist can rely on *logic* to cash out 'just false' or 'incompatible' or the like: she can use *negation* to express that α is 'just false', as $\neg\alpha$, according to such theorists, is (logically) incompatible with α. But as Priest notes, there's a clear sense in which the classical theorist does not ensure against the 'possibility' of gluts: she simply rules them out 'up to triviality', that is, she rules out all inconsistent theories shy of the limiting (and by-all-lights absurd) trivial theory.

The notion of a 'just true theory', by classical lights, comes to this: the theory is true, and it's inconsistent only if it's trivial. And for classical-logic theorists, it is *logic* that ensures the 'inconsistency at the price of triviality' ingredient. The problem of *just true*, *just false* and so on [1, 6, 7, 9, 10, 11, 14] is the challenge to glut theorists: how can *you* (viz., glut theorists) enjoy this common notion of a *just true theory* (or just-false theory, or etc). And Priest's reply, which relies on adding logical resources to elementary logical vocabulary, gives the right idea: namely, the glut theorist can add the logical-strength claim $\alpha \rightarrow \perp$ to her theory, and this serves to say that α is 'just false' in the target sense.[1] So goes the reply.

2.1 *Problem with Priest's solution*

The problem with Priest's reply is not in its spirit: the direction and idea(s) behind the reply are right. The problem is in its formulation. In particular, the connective to which Priest points – the given arrow – is very, very strong. Indeed, as Priest says elsewhere, the truth of $\alpha \rightarrow \beta$, for Priest's in-

[1] Similarly, $\alpha \wedge (\neg\alpha \rightarrow \perp)$ serves as 'just true', etc.

tended arrow and preferred semantics, requires a *logical connection* between antecedent and consequent:

> In weak relevant logics, of the kind required for paraconsistent set theory and semantics, if $\alpha \to \beta$ is true, then α entails β [i.e., has β as a *logical* consequence]. That is, α, on its own, is a logically sufficient condition for β. [13, p. 73f]

And that's the problem. Priest's reply to the just-false problem involves a *logical-strength* connection between the 'just false' antecedent and the explosive consequent. The problem is that $\alpha \to \bot$ is virtually *never* true![2] On Priest's given account, with the corresponding semantics, the glut theorist would truly say that α is 'just false' exactly when there is *no non-trivial world whatsoever* – no (non-trivial) possible or impossible world – in which α is true. That would do the trick of indicating that you take α to be acceptable on pain of accepting the trivial theory; but it's too much – much too much.

The glut theorist's solution to 'just true' (just-false, etc) ought at least afford *true* statements of 'just truth' or 'just false', etc; but Priest's solution, in the end, involves such a strong claim that it is unlikely to be true – at least given Priest's prefered (non-normal worlds, standard relevant-logic) semantics [12]. Furthermore, it is a very complicated solution, as a brief look at the details of the arrow (the logical-strength connective) involved reveals [12] – details left for the reader to explore.

2.2 *Avenues of counterreply*

There are various replies that one might give on behalf of the letter of Priest's proposal, including (most likely) fiddling with the conditional to try to get something weaker and more suitable than logical entailment [4]. But it seems to me that the core idea is simple (and viable) enough without the search for the appropriate-strength logical connectives; the essential reply to the just-true (et cetera) problem is independent of conditionals or, at least, independent of logical rules and vocabulary. I propose the following.

[2] Because we're assuming (with Priest) that logic affords the trivial model, sentence $\bot$ need not be unsatisfiable in the classical sense, but only explosive – implying all sentences. (Example: if we have a truth predicate available, $\forall x\, \mathsf{True}(x)$ is explosive.)

3 *Theory-specific,* non-logical *rules*

When classical theorists talk of their theories being *just true*, they speak only 'up to triviality'. Classical theories are true only if consistent or trivial. There's no in-between. And it's *logic*, according to classical theorists, that ensures the divide. Closing our theories under logic ensures that the theory is consistent or trivial. Such is the classical (and any explosive-logic) story.

Glut theorists, I suggest, reject (or ought to reject) the picture. Logic is more liberal than the explosive features posited by classical theorists. Close your inconsistent theory under logic and you can (and, if things go well, will) end up well shy of the trivial theory. There's nothing in *logic*, at bottom, that changes this story.

Does this mean that glut theorists cannot sensibly speak of true and consistent – or, in a standard phrase, *just true* – theories? Are classical theorists and glut theorists forever and always talking past each other? The answer, in short, is: no. But there is a basic difference. The difference, I suggest, between glut theorists and their classical counterparts is not in the function or import of *just true* (similarly, *just false*, etc) claims; it's rather in what 'grounds' the effect – what does the 'ruling out', the 'delivering triviality'. For the classical theorist, *logic* – and its 'enjoyment' of explosive operators (e.g., negation) – delivers the result; for the LP-based theorist, the effect is achieved via *non-logical* rules.[3] That, in the end, is the long and short of it. But what sort of non-logical rules?

[3] It is well-known that target glut theorists reject the existence of any explosive and exhaustive operator, as defined in [1, §3.1], calling such things 'incoherent' or etc. (See too work by Priest [12] and, though not a glut theorist, Field [8].) But such common discussions mask an important ambiguity. Glut theorists reject that there are any explosive operators in the usual sense defined via logic: O is explosive if $\mathsf{O}(A)$ and A jointly *imply* – logically imply – B, for all A and B. But glut theorists, I am suggesting, do have a sort of explosive operator, where this is understood relative to a closure operator of a given theory. When one constructs a closure operator for a theory, one can't escape logic: it's part of any closure operator (since logic is universal, ubiquitous, topic-neutral, etc.). But logic often comes up short, and one needs to add non-logical (extra-logical) rules to construct the appropriate closure operator [2]. And here, the non-logical rules can result in a closure operator that treats otherwise non-explosive operators as *explosive according to the closure operator Cl*, that is, for any A and B, $B \in \mathsf{Cl}(\{A, \mathsf{O}(A)\})$, even though B doesn't follow (via logic) from A and $\mathsf{O}(A)$.

3.1 *Shriek rules*

The classical theorist, as above, lets *logic* do the work of delivering theories that are consistent or trivial. The classical logician needn't add any rules to her theory to ensure that $\alpha \wedge \neg\alpha$ collapses the theory into triviality; logic does that. But glut theorists, I suggest, reject (or ought firmly reject) as much. If such collapse is appropriate – if the given theory is to be consistent (up to triviality) – the glut theorist must resort to *non-logical rules* to do the job. And that's what she does: adds non-logical rules – namely, *shriek rules.*

Following standard 'shriek' notation,[4] let $\alpha!$ be $\alpha \wedge \neg\alpha$. In general, then, where $\vdash$ records a rule, α's *shriek rule* takes the natural form:

$$\alpha! \vdash \bot$$

Such shriek rules, on the account I am suggesting, are not *logical rules*; they're *non-logical*, theory-specific rules motivated by the (presumed-to-be-consistent) domain in question.

The idea of non-logical rules for a particular theory is not uncommon. If one thinks that knowledge delivers truth, one thereby thinks that the theory of knowledge has the corresponding (so-called release) rule: $\mathsf{K}\alpha \vdash \alpha$. This rule is not, of course, given by *logic*; it is a non-logical rule peculiar to the theory's phenomenon (viz., knowledge). On the other hand, while non-logical rules are common in theories, shriek rules are very much unfamiliar. But's that's not surprising; many theorists (certainly many philosophers) are explosive-logic theorists who think that the shrieking is going on in the background – not something that need be explicitly mentioned. Things are different from the glut-theoretic perspective.

What motivates the *shriek rules* is the rejection that the given sentences are anything but, in a phrase, 'just true'. In explosive logics (e.g., classical, intuitionistic) such shrieking is unnecessary; it's already going on 'silently' in the *logical* rules. But glut theorists, on my account, see such shrieking as something to be explicitly added; it's not something that logic delivers; it's a theory-specific rule(s) that delivers the target 'incompatibility' – the *inconsistency at the cost of triviality* feature of the given theory.

The actual shape of shriek rules, as I shall formulate them, is tied to predicates of one's language. In particular, piggy-backing on [2], define a

[4] This is standard in paraconsistent literature, found also in Priest's discussion [12, §8.5]. One may pronounce 'α!' as 'alpha shriek'.

predicate P's shriek rule thus:[5]

$$\exists x_1, \ldots, \exists x_n (Px_1, \ldots, x_n \wedge \neg Px_1, \ldots, x_n) \vdash \bot$$

Such rules, in target sub-classical paraconsistent logics, deliver the right sort of theory of Ps, namely, a theory which is consistent or trivial: the theory is consistent or it collapses into the trivial theory upon the existence of an object with respect to which P is glutty (i.e., is both true and false of the object, the object in both the extension and antiextension of the predicate).

A theory that contains (explicit) shriek rules, so understood, is a *shrieked theory*. And these are the core of the reply to the 'just true' (similarly, 'just false', etc.) problem.

3.2 *Shrieking theories*

Let us suppose that we take a domain (or phenomenon) to be consistent; we take its true theory to be the sort of theory for which a 'just true' theory is in order. In taking the given domain to be consistent, we reject that the true theory is inconsistent; we reject that there are predicates of the theory's language that deliver gluts. But how does our theory reflect this?

The answer should now be evident: we shriek the theory by adding shriek rules for target (in the supposed case, all) predicates. Shrieking *all* predicates in the language of one's theory suffices to ensure the target result: namely, that one's theory is true only if consistent or trivial. And that's what we wanted: our theory is 'just true', and so inconsistent only if trivial.[6]

[5] In the shriek notation (using vector notation for sequence of variables and $\exists\vec{x}$ for existential closure of $\vec{x}$), the following may be abbreviated as: $\exists\vec{x}(P\vec{x}\,!) \vdash \bot$.

[6] While proofs and full formal details are left for elsewhere, the formal situation is worth waving at. Note first that, as is well-known, one can add the trivial model to classical logic without a difference (where the trivial model is the 'standard' first-order model in which the extension and anti-extension of every predicate is equal to the relevant n-fold product of the domain – basically, all predicates are true and false of all n-tuples). Hence, if one treats this model as a classical model – as something that neither classical consequence nor theoremhood 'rules out' – then the formal situation is exactly the standard classical one: M is a model of a fully shrieked theory iff M is a classical model of the theory. If one doesn't count the trivial model among the classical models (and, in the end, the choice is largely terminological), one gets the target situation in disjunctive form: M is a model of fully-shrieked theory T just if M is a classical model of T or M is the trivial model.

4 *The core reply: just shrieking*

The target problem is a challenge to glut theorists. Glut theorists enjoy a simple, unrestricted truth (and exemplification, etc) predicate at the 'cost' of an entirely non-explosive logic. Logic, as I see it, fails to rule out inconsistency – rule out 'gluttiness' at all (and not just up to triviality). The longstanding problem of *just true* [1, 6, 7, 9, 10, 11, 14] – equivalently, *just false*, *really not true*, etc – is to preserve the simplicity of an unrestricted truth (or 'exemplifies' or etc) predicate, in and for one's language, while also accommodating the apparently common notion of *just-true theories*, a notion with which falsehood (truth of negation) explodes: it's *impossible* for a falsehood to be *just true*.

Impossible? Yes, but the sense of *impossible* is not – I have suggested – a logical one: glut theorists reject (or, again, ought to reject) that *logic* rules out inconsistencies. Hence, if inconsistency is to be ruled out, it has to be ruled out in some other (non-logical) way – though only up to triviality.

The solution, in the end, is simple but powerful. When glut theorists – just as with classical theorists – claim that a (closed) theory is *just true*, they succeed in ruling out gluttiness up to triviality; they imply that it's either (negation-) consistent or that it's trivial. The difference between glut theorists and their classical counterparts is only in the 'deliverer of triviality', not in the import of 'just true'. Classical theorists see logic (its rules and axioms) as the silent deliverer, while glut theorists see the need for going beyond logic in the form of theory-specific shriek rules. But that's it.

If there is a 'just true' (similarly, 'just false', etc) problem for glut theorists which is not addressed by the given shrieking-theories reply, I hope that the current paper serves as a challenge to formulate it. Until then, the sound of shrieking shall – and must – go on.[7]

References

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