

# 9

## Non-detachable Validity and Deflationism

*Jc Beall*

### 9.1 Introduction: History and Setup

This chapter began as a paper in St Andrews on validity and truth preservation, focusing on a point that I (and others) had observed: namely, that validity is not truth preserving in any *detachable* sense (to be explained in the chapter). The paper was later expanded for a conference in Princeton on the philosophy and logic of truth (and their interplay): one's views on validity can often be constrained by one's philosophy of truth (or allied notions). The chapter before you, which is a lightly modified version of the later conference presentation, focuses on one instance of such interplay: deflationism about truth and the issue of (non-) 'detachable validity'. My chief aim in the chapter—as in the talks that occasioned it—is simply to raise the issues rather than decisively answer them. With this aim in mind, I have attempted to leave this contribution in its 'talk form', highlighting only the essential points of the discussion, expanding only where clarity demands it, and often using bullets instead of paragraph form.

#### 9.1.1 *What is deflationism about truth?*

Too many things. Deflationism about truth is not one but many views, united only by the thought that truth plays no explanatory role: truth (or 'true') serves as a vehicle for explanations (or, generally, generalizations) of the world, not as explaining anything in the world. Instead of trying to untangle all such views, I shall focus on the 'transparency' version of deflationism about truth, a strand of Quinean 'disquotationalism' whereby 'true' is a *see-through* device brought into the language (for the language) of practical necessity—the in-practice necessity of expressing long generalizations (e.g. 'everything in such-n-so theory is true', etc.).

This sort of ‘transparency’ view goes back to Quine (1970), was clarified by Leeds (1978), and in turn was widely advanced by Field (2001) as a ‘pure’ version of disquotationalism. My focus on this version of deflationism is not to suggest that others aren’t important or that this one has the best chance of being true. I focus on it because it has some common intuitive appeal, and, besides, I’ve thought more about it than others, and advanced a version (Beall 2009).

### 9.1.2 *Main issue of the chapter*

The issue concerns validity and arises out of (not unfamiliar) truth theoretic paradoxes—particularly, curry paradox.<sup>1</sup> In what follows, I review a result to the effect that validity is not truth-preserving in any ‘detachable’ sense, spelling this out in terms of what this means for one’s validity predicate or corresponding validity connective. In short: for curry-paradoxical reasons, there’s no valid argument from the validity of an argument and the truth of its premises to the truth of its conclusion. Saving some sense in which validity detaches is a goal that motivates a stratified or hierarchical approach to validity (Myhill 1975; Whittle 2004). But, details of the stratified approach aside, a question concerning deflationists about truth immediately emerges: can *deflationists* about truth go stratified about validity? I briefly discuss this question and, relying on a proposal by Lionel Shapiro (2011), briefly suggest an affirmative answer.

## 9.2 Background Conception of Truth

The aim of this section is simply to set some terminology. The background ‘transparency’ conception of truth is along the lines mentioned above: our truth predicate is an expressive device and nothing more; it was not introduced to name an important or otherwise explanatory property in the world; it was brought into the language to serve as a vehicle for explanations or, more generally, the expression of generalizations.

### 9.2.1 *Transparent or see-through predicate*

- Let  $\ulcorner \urcorner$  be some naming device over language  $L$ , some function that assigns each sentence  $A$  an appropriate name  $\ulcorner A \urcorner$ . (This may be via a suitable quotation convention, as in many natural languages, or may be something fancier, such as Gödel coding.)

<sup>1</sup> I use ‘curry’ in ‘curry paradox’ as a predicate that classifies various paradoxes that, while not exactly like Curry’s original paradox (arising from combinatorial logic), are clearly of the type to which Curry originally pointed (Curry 1942).

- Let  $\varphi(x)$  be a unary predicate in L.
- Let  $\widehat{A}$  be the result of substituting  $\varphi(\ulcorner B \urcorner)$  for all (non-opaque) occurrences of B in A.
- $\varphi(x)$  is said to be a *transparent* or *see-through* predicate for L just if A and  $\widehat{A}$  are equivalent for all A in L.

Example: assuming that negation is non-opaque,  $\varphi(x)$  is transparent for L only if  $\varphi(\ulcorner \neg A \urcorner)$  and  $\neg\varphi(\ulcorner A \urcorner)$  are equivalent, and similarly only if  $A \odot B$  and  $\varphi(\ulcorner A \urcorner) \odot B$  are equivalent for any (non-opaque) binary connective  $\odot$ , and so on. Throughout, ‘equivalent’, at the very least, involves *logical equivalence*, so that, for any A in the language, A and  $\widehat{A}$  are (at least) logically equivalent if  $\varphi(x)$  is transparent for L.<sup>2</sup>

### 9.2.2 Transparency conception

With the notion of a transparent or see-through predicate (or ‘device’) in hand, the transparency conception (or view) of truth may be characterized as follows.

- The transparency view has it that truth is a (logical) property expressed by a see-through device in (and for) our language.
- *Truth* is in-principle-dispensable but in-practice-indispensable: God could fully specify our world without using the truth predicate; but our finitude requires that we use it—for familiar purposes of generalization.
- Truth, on this conception, is not at all an explanatory notion; it is involved in explanations in the way that our ‘voice box’ is involved: we use it to express the explanation, but the explanation doesn’t itself invoke it. (All of this is standard ‘deflationary’ story.)

The crucial negative point is that truth is not an important explanatory notion—not explanatory at all. And this view spills over into other common notions: satisfaction and denotation, as is well known, must be treated along similar deflationary lines.

What about validity? *Must* it too receive a ‘deflationary’ philosophy? Against some (Shapiro 2011), I think not; but I will not engage on this issue here. The question I shall briefly address is twofold:

<sup>2</sup> By *logical equivalence* is meant *whatever, in the end, the given logic—validity relation—counts as equivalent*. If  $\vdash$  is the logic, then A and B are logically equivalent, in the target sense, just when  $A \dashv\vdash B$ . (NB: what’s important is that a logic—and, in particular, the notion of logical equivalence—be understood broadly enough to allow for the target notion of transparent predicates. The resulting logic may—and, in truth theory, generally will—involve rules governing special predicates. Think, for example, of identity or, more on topic, truth. My interest is not in whether these are ‘really’ logical expressions.)

- How, if at all, might deflationists about truth maintain that *validity* is ‘detachable’ (in a sense to be explained below)?
- How might deflationists about truth be similarly deflationary about (detachable) validity?

By way of answers, I shall suggest—though only suggest—a marriage of ideas already available: one from John Myhill (1975, 1984) and the other, more recent, from Lionel Shapiro (2011). But first I rehearse a (perhaps now-familiar) point about validity and truth preservation, and I make explicit a corollary concerning the (non-) ‘detachability of validity’.

### 9.3 Predicates and Connectives

I assume, throughout, that we can take any binary (indeed, n-ary) sentential predicate—that is, a predicate defined over all sentences—and get an equivalent corresponding sentential operator:

$$A \odot B := \Pi(\ulcorner A \urcorner, \ulcorner B \urcorner)$$

Given a (transparent) truth predicate, one can go the other way too; but the predicate-to-operator direction is the important one for current purposes.<sup>3</sup>

### 9.4 Validity, Truth Preservation, and Detachment

It’s known that, for *curry-paradoxical* reasons, transparent truth theorists need to reject that validity is ‘truth preserving’ in any *detachable* sense (Beall 2006, 2009; Field 2008; Priest 2006); they need to reject that there’s a valid argument from the validity of arbitrary argument  $\langle A, B \rangle$  and the truth of  $A$  to the truth of  $B$  (Beall 2009, 35ff). Let me make this plain.<sup>4</sup>

- Curry sentences arise in various ways, commonly via straightforward ‘curry identities’ such as  $c = \ulcorner \text{Tr}(c) \odot \perp \urcorner$ . (Example: in English, one might have it that ‘Bob’ denotes ‘If Bob is true then everything is true’, thereby grounding a suitably necessary link between ‘Bob is true’ and the given conditional.)

<sup>3</sup> The other direction relies explicitly on the truth predicate  $\text{Tr}(x)$ . In particular, where  $\odot$  is a binary sentential operator, one defines the corresponding predicate  $\Pi$  by setting  $\Pi(\text{Tr}(x), \text{Tr}(y))$  to be true just when  $\text{Tr}(x) \odot \text{Tr}(y)$  is true.

<sup>4</sup> While I wave at examples, I assume familiarity with curry paradox and recent debate on it. Also,  $\perp$  is any *explosive* sentence—a sentence implying everything.

- *Detachable*. A binary connective  $\odot$  is *detachable* iff the argument from  $A \wedge (A \odot B)$  to  $B$  is *valid*, iff the argument from  $\{A, A \odot B\}$  to  $B$  is valid.<sup>5</sup>
- For curry-paradoxical reasons, transparent truth theorists need to reject the validity (or even unrestricted truth) of ‘pseudo modus ponens’ or PMP (Beall 2009; Restall 1993; Shapiro 2011) for *any detachable* connective  $\odot$ .<sup>6</sup>

PMP.  $A \wedge (A \odot B) \odot B$

- |  |  |
|--|--|
| 1. $c = \ulcorner \text{Tr}(c) \odot \perp \urcorner$ .  | [Empirical fact (let us say)]              |
| 2. $\text{Tr}(c) \wedge (\text{Tr}(c) \odot \perp) \odot \perp$ .                              | [PMP]                                      |
| 3. $\text{Tr}(c) \wedge \text{Tr}(\ulcorner \text{Tr}(c) \odot \perp \urcorner) \odot \perp$ . | [2; Transparency]                          |
| 4. $\text{Tr}(c) \wedge \text{Tr}(c) \odot \perp$ .  | [1,3; Identities]                          |
| 5. $\text{Tr}(c) \odot \perp$ .  | [4; Substitution of $A$ for $A \wedge A$ ] |
| 6. $\text{Tr}(\ulcorner \text{Tr}(c) \odot \perp \urcorner)$ .                                 | [5; Capture/Transparency]                  |
| 7. $\text{Tr}(c)$ .  | [1,6; Identities]                          |
| 8. $\perp$ .   | [5,7; MP—i.e., $\odot$ -detachment]        |

*Question:* how does the rejection of PMP for any detachable connective amount to the rejection of the detachability of validity? As follows:

- Suppose, now, that in addition to a truth predicate, we have an adequate *validity predicate*  $\text{Val}(x, y)$  in our language. Then to say that  $\odot$  is detachable is to say that  $\text{Val}(\ulcorner A \wedge (A \odot B) \urcorner, \ulcorner B \urcorner)$  is true.
- Define a corresponding validity *operator*:  $A \Rightarrow B := \text{Val}(\ulcorner A \urcorner, \ulcorner B \urcorner)$ . Then an operator  $\odot$  is detachable just if  $A \wedge (A \odot B) \Rightarrow B$  is true.
- But now the point is plain: validity  $\Rightarrow$  is detachable iff  $A \wedge (A \Rightarrow B) \Rightarrow B$  is true iff PMP holds for the validity operator.
- Hence, validity itself is not detachable.<sup>7</sup>

*Parenthetical note.* I am embarrassed to say that I didn’t sufficiently spell out this point in my *Spandrels of Truth* discussion (Beall 2009, ch. 3), though did spell it out enough for it to be an implication: ‘I reject that valid arguments are ttruth-preserving in anything beyond the hook sense’ (i.e. in any detachable sense) (Beall 2009, §2.5, 37), and ‘one can know that an argument is valid and know that its premises are all ttrue, but nonetheless remain without a valid argument that takes one from such information [i.e. truth of premise and validity of argument] and

<sup>5</sup> I assume throughout that conjunction  $\wedge$  is normal. Giving a non-standard account of  $\wedge$  affords options—but I won’t here look into that. (Similarly with respect to standard structural rules: I assume them, and do not here discuss giving them up.)

<sup>6</sup> For readability’s sake, let  $\wedge$  bind more tightly than  $\odot$ , so that  $A \wedge B \odot C$  is  $(A \wedge B) \odot C$ .

<sup>7</sup> To use Restall’s lingo (Restall 1993) now common in this area: transparent truth theorists need to be ‘really contraction free’ to avoid curry paradox—and this, I’m here noting, applies to *all* binary connectives (including the validity one). See Shapiro (2011) and Beall and Murzi (2013).

the [non-detachable sense of validity truth-preservation] to the given conclusion' (Beall 2009, 36). Only recently, after having returned to some of these issues in a paper with Julien Murzi (Beall and Murzi 2013), did I see things perfectly simply and clearly in the way I've laid it out above. Looking at the literature, it is clear to me that Lionel Shapiro (2011) was the first to make explicit what was nearly—but only nearly—explicit in my claim above, and I probably owe my appreciation of the point to him. (I briefly discuss some of his key work below.) But other work, cited in Beall and Murzi (2013), is also in the area—perhaps most explicitly John Myhill's (1975) and Bruno Whittle's (2004).

## 9.5 Validity: Detachable via Stratification?

Some might think that detachable truth preservation or, as I'll just say here, the detachment of validity is essential to our notion of validity. Suppose that that's right. How, then, are we to keep detachment without falling prey to the perils of PMP?<sup>8</sup>

The most natural thought points to a stratified or hierarchical notion of validity. For precisely the sort of reasons above (though put in different ways), John Myhill (1975) proposed that validity be understood along a stratified front, as did Bruno Whittle (2004) more recently. The idea, in a nutshell, is that we have no cover-all validity relation but many limited relations—or, if you want, we have one big stratified relation, with each stratum itself a validity relation. This way, we can have that each validity relation is truth preserving (and, so, detachable); we can truly say that validity<sub>i</sub> is 'detachable' by using some 'higher' (or extended, or etc.) relation:

$$A \wedge (A \Rightarrow_i B) \Rightarrow_{i+1} B$$

But a question arises: namely, whether any such stratified approach to validity is philosophically compatible with our target sort of deflationism about truth.

## 9.6 Compatible with Deflationism?

Is the stratified approach available to *deflationists* about truth deflationism—particularly, the sort of 'merely expressive device' ones at issue? I don't see why not. Moreover, I think that there's a clear path towards taking an expressive-device deflationary view of validity—a path cut recently by Lionel Shapiro (2011). Shapiro,

<sup>8</sup> I should note that, in recent work (Beall 2012), I have come to think that our language is entirely detachment-free—containing no detachable connectives (hence, no detachable validity connectives). I cannot go into these ideas here, and suppress them throughout. I think that the issues raised in the current paper are still very much worth putting forward for exploration and debate—my main aim in this chapter.

I should make plain, agrees that validity is non-detachable in the given sense.<sup>9</sup> While he does *not* consider stratified validity, Shapiro's idea for a way to see validity as 'deflationary' applies just as well in the stratified case. Let me present the basic idea, and then summarize its relevance here.

Shapiro's paper is rich with ideas, but I shall focus on only one thing. For present purposes, what Shapiro gives us is a sense in which the validity *predicate*—versus operator—may be seen as an expressive device, a generalizing device along the lines of truth. Importantly, Shapiro's picture is one in which we already have validity operators in the language, and we introduce a validity *predicate* to generalize over them. And this, on the Shapiro picture, is precisely what is going on with other expressive devices like the truth predicate and similarly falsity predicate. How does this go?

Invoking an analogy from Anderson and Belnap (1975), Shapiro's idea is strikingly simple. For convenience, let me set some terminology:

- A *negation* is a sentence whose main connective is negation.
- A *nullation* is a sentence whose main connective is the null operator. (Every sentence is a nullation.)

In turn, we are to see 'true' as generalizing over nullations in the same way that, for example, 'false' generalizes over negations: on a transparency conception,  $\text{Tr}(\ulcorner A \urcorner)$  and  $A$  are intersubstitutable in the way that  $\text{False}(\ulcorner A \urcorner)$  and  $\neg A$  are. (On a transparency conception, falsity is generally the transparent truth of negation:  $\text{False}(\ulcorner A \urcorner)$ , by definition, is  $\text{Tr}(\ulcorner \neg A \urcorner)$ , equivalently  $\neg \text{Tr}(\ulcorner A \urcorner)$ .)

Suppose, now, that we have an entailment or validity connective  $\Rightarrow$  in the language, and let an *implication* be a sentence with  $\Rightarrow$  as its main connective. Shapiro argues that a validity predicate generalizes over implications in exactly the way that the truth and falsity predicates generalize over nullations and negations, respectively. That's the basic idea. In a picture:

- 'true' generalizes over nullations;
- 'false' generalizes over negations; and
- 'valid' generalizes over implications.

In our stratified setting, we simply broaden the point about validity:

- 'valid<sub>i</sub>' generalizes over implications<sub>i</sub> (i.e.  $\Rightarrow_i$  claims).

<sup>9</sup> He explains this by going 'really contraction-free'—a logic that, unlike leading transparency theories, gives up *substructural* contraction, and thereby the PMP form of contraction. I will slide over these details for present purposes.

An example: *the argument from  $A \wedge (A \Rightarrow_i B)$  to  $B$  is  $\text{valid}_{i+1}$* . More ordinary examples, using ‘consequence’ instead of ‘validity’ (as the former has a more ordinary ring), are claims such as *Axiom 1 of So-n-so’s theory is a consequence of something the Pope said*. Here, the validity (or consequence) predicate is generalizing over implications in a familiar way: either the Pope said  $x$  and *that  $x$  is true entails that Axiom 1 is true* or the Pope said  $y$  and *that  $y$  is true entails that Axiom 1 is true* or . . . so on. (Here, I use ‘true’ in its usual see-through role, just for convenience. This can be dropped.) Along the same lines: *everything in theory T is a consequence of something in theory T’*. And so on.

The examples themselves may be less important than the main point here: namely, that this provides at least one clear sense in which validity predicates—even if stratified (as we’re assuming)—can be seen as expressive devices (generalizing devices) along the same front as ‘true’. The sole role of an expressive device is to generalize over some fragment (possibly improper fragment) of the language; and the device achieves its role in virtue of simple rules (e.g. ‘capture and release rules’ or ‘intro and elim rules’, etc.)—and we needn’t read into the basic rules any ‘metaphysical baggage’, but instead can see such devices as merely logical. Validity predicates can be seen as such—stratified or not.

## 9.7 Questions and Replies

But let me quickly answer a few questions, before summarizing and closing.

\* *Question.* But this approach to ‘deflationism about validity’ only works if we already have validity operators in the language. How is ‘validity’ (the predicate) then seen as on par with our so-called *merely see-through device* ‘true’?

\* *Answer.* There are differences: the truth one is in-practice indispensable, while the others aren’t (ignoring propositional quantification). For example, we can generalize over negations and implications using only ‘true’ in its standard role. If we get rid of ‘true’, we’d be stuck again—regardless of whether ‘valid’ can do its generalizing role. But all that this shows is that truth is indispensable in a way that, so long as we have truth, ‘valid’ and ‘false’ aren’t; it doesn’t undermine—as Shapiro himself notes—that the sole role of the given *predicates* is the given generalizing work (even if that work can be done by other devices in the language).

In short, we can clearly acknowledge, along the Shapiro picture, that all of ‘true’, ‘false’, and ‘valid’ (the predicates) are *expressive devices*, generalizing devices that do their job via their basic rules (e.g. Release and Capture, or Intersubstitutability, or some such). But we can also distinguish between *in-practice-indispensable* ones



and *in-practice-useful* (or the like) ones. Truth, on the transparency view, is in the former category, and the others—stratified or not—along the latter.<sup>10</sup>

\* *Question.* If you go stratified for ‘validity’, why not also for ‘true’? There may not be an *incompatibility* between transparency about truth and stratified validity, but if you go stratified for one, why not for both?!

\* *Answer.* The expressive role of ‘true’ requires more than what that of ‘valid’ may require. You can’t generalize over all sentences (all nullations) in your own language with stratified truth. (Indeed, if we focus on the Shapiro picture of *expressive device*, it’s plain that *validity* is a notion for which stratification makes sense, much like negation itself. But the *null operator* can’t be stratified: this marks again the special status that truth enjoys.)

## 9.8 Summary

There’s a wide variety of deflationary views, perhaps each with its own peculiarities and problems and virtues. I’ve focused on the *transparency view*, one to which ‘true’ is nothing more than an expressive device—a full *see-through* device over one’s entire language. One issue topic of fundamental concern is *validity* in a transparent-truth setting. I’ve argued that such theorists need to reject that validity is detachable. A natural way towards *some* sense of ‘detachable validity’ is via stratification. I’ve suggested that stratification is *not* philosophically incompatible with an appropriate device-deflationary view of ‘valid’, and waded at Shapiro’s approach as one way (probably among others) to see stratified validity predicates as ‘expressive devices’. There are no doubt lots more issues worth thinking about with respect to both validity and truth, but my aim has been to highlight at least one—and I hope I’ve done that.<sup>11</sup>

<sup>10</sup> Accordingly, I think that, contrary to Shapiro (2011), transparent-truth theorists needn’t be similarly deflationary with respect to ‘validity’. But I do think that they can be, and indeed can maintain that validity—qua stratified notion—is both detachable *and* ‘deflationary’ in the sense discussed.

<sup>11</sup> I am grateful to the organizers of the FLC workshops and conference(s) in St Andrews that occasioned many of the ideas in this chapter. I’m very grateful to many people for discussion, in one form or another, on many different occasions, including (but probably not limited to) Alexis Burgess, John Burgess, Andrea Cantini, Colin Caret, Roy Cook, Aaron Cotnoir, Hartry Field, Michael Glanzberg, Ole Hjortland, Hannes Leitgeb, Vann McGee, Julien Murzi, Charles Parsons, Graham Priest, Agustin Ráyo, Stephen Read, Greg Restall, David Ripley, Marcus Rossberg, Josh Schechter, Lionel Shapiro, Bruno Whittle, and many participants at the ‘Pillars of Truth’ conference in Princeton.

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