A neglected response to the Grim result

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1. The Grim result

Is there a set of all truths? According to a familiar argument by Patrick Grim the answer is 'no'. Grim's argument is a reductio which runs as follows.[1]

Suppose, for *reductio*, that there is a set of all truths; call it 'T'. Let $\mathcal{P}(T)$ be the powerset of T. To each element s_i of $\mathcal{P}(T)$ there exists a truth. For example, to each s_i corresponds the following truth

$$s_i \quad \mathcal{P}(T)$$

Alternatively, let t_1 be a truth. Then for every element s_i of $\mathcal{P}(T)$, one of the following is a truth

$$t_1 \quad s_i$$
$$t_1 \quad s_i$$

But, then, there are at least as many elements of T as there are elements of $\mathcal{P}(T)$, which contradicts Cantor's theorem. Hence, there is no set of all truths.

The Grim result, then, is that there is no set of all truths. This result is thought by some (many) to be *strongly* counterintuitive. But if the result is to be rejected then a flaw in Grim's argument must be found. Whilst many philosophers have attempted to find such flaws nobody has yet to question Grim's main premiss, namely

(1) To each element of $\mathcal{P}(T)$ there corresponds a (unique) truth.

How might one go about denying (1)? I will briefly discuss a natural but neglected option.[2]

2. The S-view and necessary truths

In light of Grim's examples (1) appears to be perfectly obvious, which is perhaps why nobody has challenged it. Nonetheless, there is a familiar theory of propositions which directly challenges (1) – namely, that propo-

[1] Grim's argument and relevant discussion can be found in a number of *Analysis* papers, including Grim 1984. 1986, 1989, 1990. A more extensive discussion can be found in Grim 1991, from which I draw in §3 of the current paper.

[2] I ignore so-called dialetheic responses to the Grim result, given that most philosophers are more interested in consistent solutions to the problem. However, I should note that I am generally very sympathetic with to dialetheism.

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sitions are functions from worlds to truth values. For ease of reference call this view the S-view.[3]

If the S-view is accepted then Grim's argument is immediately blocked. On the S-view there is precisely one necessary truth; for there is exactly one function from all worlds to The True. Accordingly, the S-view is in position to grant that there are truths for every element of $\mathcal{P}(T)$; however, the S-view will deny that the cardinality of such truths equals that of $\mathcal{P}(T)$. On the S-view, the cardinality of the given truths, given that such truths are necessary, is 1. The flaw in Grim's argument is that he miscounted.

In so far as (1) is supported only by necessary truths the S-view provides a quick response to the Grim result: Since there is exactly one necessary truth (1) is false. The question, then, is whether (1) can be established by appeal to contingent truths. Grim argues that the answer is 'yes'.

3. *Contingent version?*

Grim argues that (1) can be established by appeal to contingent truths. If he is right, then the response I have suggested falls short of undermining the Grim result; in that case it undermines only the claim that there is no set of necessary truths. The question, then, is whether Grim is right about a contingent version. I will argue that he is not right.

Grim's construction of the requisite contingent truths is straightforward.[4] Let *c* be any contingent truth. Let s_i be an element of $\mathcal{P}(T)$. Then, claims Grim, one of the following is a contingent truth:

$$c \quad (c \quad s_i)$$
$$c \quad (c \quad s_i)$$

That these are contingent follows from the fact that, though each of $(c \quad s_i)$ and $(c \quad s_i)$ is necessary, the conjunction of a contingency with a necessity is a contingency.

The question is whether (1) is established by such contingent truths; that is, the question is whether Grim's construction establishes a unique truth corresponding to each element of $\mathcal{P}(T)$. The answer, given the S-view, is 'no'. Let $\top$ be the necessary truth, and let *c*, as above, be any contingent truth. Then

$$c \quad \top$$

is necessarily equivalent to

$$c$$

3 'S' in 'S-view' is less for sets than for Robert Stalnaker, who has been one of the chief defenders of the S-view of propositions. I assume familiarity with the S-view.

4 The construction may be found in Grim 1991: 94–95.

which is to say that '$c \quad \top$' and 'c' express the same truth – the contingent truth, c. But given that

$$c \quad s_i$$

is a necessary truth (if true), it follows that, on the S-view, Grim's construction

$$c \quad c \quad s_i$$

is the same contingent truth (if true) as c. (Similarly if c is false, etc.) Accordingly, Grim's construction yields only the contingent truths with which we began; the construction, however, gives no reason to think that such truths outrun, or even match, the cardinality of $\mathcal{P}(T)$. For this reason, Grim's attempt to support (1) by appeal to contingent truths fails.

If the foregoing is correct then the Grim result has been undermined. Grim's argument against the existence of a set of all truths founders on a miscount of the truths involved. That there is a miscount involved follows from the S-view of propositions. At this stage, however, some will object that the S-view itself faces well-known difficulties.

4. The S-view and the Grim result

The S-view faces well known objections. The biggest difficulty is that, like the Grim result itself, the S-view seems to carry strongly unintuitive consequences. Indeed, that there is exactly one necessary truth strikes many as being so remote from intuition that the S-view is dropped right there.[5]

Whilst I think that there are viable defences against such objections, this paper is not the place for such defence; that is a task for another time. For now, however, I note one point.

Beyond its other virtues,[6] the S-view has the important virtue of resolving the Grim result – or Grim's paradox, as some have called it.[7] Moreover, the S-view resolves the paradox in a clear and principled way. No other theory of propositions so deftly deals with the Grim result. For this reason the Grim result may well serve to give philosophers reason to consider the S-view more carefully. If nothing else, however, this paper indicates that more needs to be said in defence of Grim's premiss (1). Counting truths is not always as easy as it may seem.

[5] Other well known difficulties arise when the S-view is combined with the claim that propositions are the (sole) content of intentional states. A consequence of this is that we are logically omniscient – we know all consequences of what we believe. See Stalnaker 1984 for further discussion and avenues of defence.

[6] As discussed, for example, by Stalnaker op. cit.

[7] Plantinga 1993, for example, calls the Grim result a paradox.

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