Negation's Holiday: Aspectival Dialetheism

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Abstract: What does the Liar teach us about English? According to 'orthodox' dialetheism, as espoused by Graham Priest (using his *LP*-based logic), the Liar teaches us that the negation of some true English sentence is true (and, hence, that English is underwritten by a paraconsistent logic). That lesson, in addition to being very simple, avoids the familiar expressive problems that confront its ('consistency') rivals. I am inclined to accept dialetheism, although not the version advanced by Priest. Liar-like sentences are true and false; however, they are also sentences in which negation is on holiday, in a sense to be explained. *Negation*, I suggest, exhibits a 'double-aspect' — behaving 'classically', for the most part, but very non-classically (indeed, 'free-floating') when involved in paradoxical constructions. Some (many) philosophers think that the very meaning of 'falsity' rules out dialetheism; the double-aspect hypothesis has a nice explanation of such thinking, and, indeed, acknowledges a sense in which it is correct. In addition, the double-aspect view avoids recent objections (by Field and Shapiro) against 'orthodox' dialetheism. In this paper I present a novel version of dialetheism — 'double-aspect dialetheism', for want of a better name. For space considerations, I assume familiarity with dialetheism and its various virtues, including the 'semantic self-sufficiency' that it affords.

1 Falsity and the Liar

What is falsity? The standard answer is that falsity is truth of negation, that falsity reduces to truth and negation; the idea is that 'is true' is our only primitive semantic predicate, that truth is our only primitive semantic 'truth value'.

In this paper, I want to take that idea seriously (that truth is our only primitive semantic value); but I also want to take dialetheism seriously — the idea that some truths have true negations. If we take both of those ideas seriously we get not *LP* (the logic usually associated with dialetheism) but, rather, a different paraconsistent logic that, as I will explain, reflects a 'double aspect' theory of negation.

The structure of the paper is as follows. §2 rehearses (relational) classical semantics, wherein truth is the only primitive truth value. In §3 I (briefly) discuss the familiar dialetheic lesson of the Liar, which calls for dropping the 'exclusion' constraint of classical semantics.[1] §4, in turn, presents the semantics (and theory of negation) that results from dropping the exclusion constraint of classical semantics. The theory of negation in §4, as will be clear, is hard to accept on its own, and, for reasons discussed in §5, I suggest that it not be accepted on its own. My suggestion, as presented in §6, is that negation enjoys a 'double aspect', behaving classically in most (familiar) cases but behaving non-classically when negation is involved in paradoxical sentences. The 'double aspect' theory of negation may be modeled via a non-monotonic (adaptive) logic that I present in §6. Finally, §7 mentions a few advantages that the 'double aspect' theory has over *LP*, especially concerning the notion of 'true only' or 'non-dialetheia'.

[1] I assume familiarity with the virtues of a dialetheic response to paradox, especially with respect to familiar 'revenge' and 'expressive' problems that confront all its many rivals. A basic review of dialetheism is given by Sainsbury [11] and a thorough review is given by Priest [8]. Beall and van Fraassen [3] provide an elementary review of *FDE* and *LP*, which are typical, indeed 'orthodox', logics associated with dialetheism.

2 Relational Classical Semantics

As above, we want to take the 'reduction of falsity' seriously; accordingly, we want to recognize no primitive semantic values (truth values) beyond truth. In turn, we want to define falsity in terms of our primitive semantic value (truth) and negation. The task can be done by using 'partial functions' for our valuations.[2] For present purposes, I will use the more general notion of a relation — leaving its functional character (if one) to be a matter of additional constraints.

2.1 Syntax

I'll concentrate on the syntax of classical propositional logic with p_i ($i \in \mathbb{N}$) being any atomic (propositional parameter) and connectives $\sim$, $\wedge$, and $\vee$. Let $\mathcal{S}$ comprise all sentences.

2.2 Valuations

A *relational valuation* (henceforth, valuation) on $\mathcal{S}$ is a relation $\mathcal{R} \subseteq \mathcal{S} \times \{1\}$. Define *exhaustive* and *exclusive* valuations thus:

» An interpretation $\mathcal{R}$ is *exhaustive* iff for any A, either $\langle A, 1\rangle \in \mathcal{R}$ or $\langle {\sim}A, 1\rangle \in \mathcal{R}$.

» An interpretation $\mathcal{R}$ is *exclusive* iff no A is such that $\langle A, 1\rangle \in \mathcal{R}$ and $\langle {\sim}A, 1\rangle \in \mathcal{R}$.

Further terminology will be useful:

» A is *true in* $\mathcal{R}$ iff $\langle A, 1\rangle \in \mathcal{R}$.

» A is *false in* $\mathcal{R}$ iff $\langle {\sim}A, 1\rangle \in \mathcal{R}$.

[2] In mathematics, a partial function on $\mathbb{N}$ is a function that is undefined for some elements of $\mathbb{N}$. If we define the *domain* of a function f to comprise all elements for which f is defined (i.e., gives a value in its co-domain), then, by analogy with mathematics, a partial function f from $\mathcal{X}$ into $\mathcal{Y}$ is such that the *domain* of f is a proper subset of $\mathcal{X}$. That is what I mean by 'partial function' here.

- » $\mathcal{R}$ *satisfies* Γ iff every element of Γ is true in $\mathcal{R}$.
- » $\mathcal{R}$ *satisfies* A iff $\mathcal{R}$ satisfies $\{A\}$.
- » $\mathcal{R}$ is a *model* of Γ iff $\mathcal{R}$ satisfies Γ.
- » $\mathcal{R}$ is a *model* of A iff $\mathcal{R}$ satisfies A.

For simplicity I will sometimes write '$\mathcal{R} \vDash A$' to abbreviate 'A is true in $\mathcal{R}$', and hence '$\mathcal{R} \vDash {\sim}A$' for '$A$ is false in $\mathcal{R}$'.

2.3 *Admissible valuations and classical consequence*

Classical semantics defines *admissible valuations* to be any valuation $\mathcal{R}$ such that

- » $\mathcal{R}$ is exhaustive. (Call this *Exhaustion*.)
- » $\mathcal{R}$ is exclusive. (Call this *Exclusion*.)
- » $\mathcal{R} \vDash A \wedge B$ iff $\mathcal{R} \vDash A$ and $\mathcal{R} \vDash B$.
- » $\mathcal{R} \vDash A \vee B$ iff $\mathcal{R} \vDash A$ or $\mathcal{R} \vDash B$.

Classical consequence $\Vdash_c$, in turn, is defined as usual:

- » $\Gamma \Vdash_c A$ iff every model of Γ is a model of A.

Logical truth (valid sentence) may be defined as truth in every (admissible) valuation: $\mathcal{R} \vDash A$. Given classical semantics, an equivalent account is available: A is logically true iff $\emptyset \Vdash_c A$.

3 *The dialetheic response to paradox*

Classical semantics, as above, allows us to treat truth as our only primitive semantic value ('truth value'); and classical logic, at least on the surface, seems to get things right, at least with respect to conjunction, disjunction, and even negation. On the other hand, various phenomena call Exhaustion and Exclusion into question. For example, vagueness seems to question the Exhaustion constraint, while the Liar seems to question the Exclusion constraint. For present purposes I will ignore the former issue; I will concentrate on Exclusion.[3]

What does the Liar teach us about English? According to dialetheism, the lesson is that some truths have true negations; in particular, 'this sentence is not true' is true and false — both it and its negation are true. Exclusion, according to the dialetheic lesson (as I advance it), needs to be rejected; there are some (admittedly peculiar) sentences that are both true and false.

While it strikes many philosophers as radical (in some pejorative sense), I think that the dialetheic lesson is quite natural. One immediate advantage of dialetheism is that it allows us to retain the foundational feature of (naive) truth: namely, the intersubstitutivity of A and $\mathsf{T}\underline{A}$, where T is our truth predicate (or, at least, our simple disquotational truth predicate). Non-dialetheic responses to the Liar are either forced to give up such intersubstitutivity or they achieve as much by reducing expressive power — requiring a richer meta-language, and so on.

The general problem, in short, runs thus: One's aim is a theory of how truth (or 'true') behaves in English — its logical behavior. In giving one's theory one attempts to avoid inconsistency by introducing some crucial semantic notion (context, stages, gaps, stability, or the like). On pain of inconsistency, one is forced — the 'revenge' problem — to say that the crucial semantic notion(s) is not expressible in the 'object language'.[4] The trouble is evident: What language

[3] I'm inclined to think that there are 'gappy' sentences; however, I also think that we consistently assert that A is gappy — that A is neither true nor false — by invoking a gap-closing negation, in which case the issue of Exclusion arises again. For present purposes, I leave gaps aside. (Note that, as is familiar, gaps alone do not resolve the Liar phenomena.)

[4] In the case of popular contextual theories, the expressibility problem is that one

was used to express the theory itself? The answer, of course, is *English* — a language that can express such notions. One's efforts to avoid the claim that English is inconsistent force one into denying what seems very difficult to deny: that English can express the crucial semantic notions.

Some philosophers, of course, may be happy to 'just live' with the noted (expressibility) tensions; I am not suggesting that such tension serves as a knockdown argument against non-dialetheic responses to the Liar. By my lights, the dialetheic response is simpler, and ultimately more natural than denying the apparent expressive power of English.

The advantages of a dialetheic response to paradox have been well-documented by Graham Priest [8], and I will not review them further here. One point is worth emphasizing: that the sorts of sentence that are true and false are peculiar ones that arise merely out of grammatical necessity. There is no reason to think that anything but such peculiar, circularity-ridden sentences are true and false. Indeed, there is no reason to think that classical logic, and in particular its theory of negation, is incorrect *except* for a 'few' peculiar sentences that enter the language due only to grammatical necessity. I will return to that point. For now, I turn to further aspects of the dialetheic lesson (at least as I see it).

4 *Dropping Exclusion: The Logic P*

Let *P* (for paradox) be the logic that results from dropping only Exclusion from the classical semantics in §2 while adding Double Negation:

» $\mathcal{R} \models A$ iff $\mathcal{R} \models {\sim}{\sim}A$.

cannot quantify over contexts, or stages, or etc., at least not to the extent that one can in the 'metalanguage'. Representative theories and their respective expressibility problems are discussed in (among other places) [6, 7, 8, 13].

The resulting logic is paraconsistent: arbitrary B does not follow from arbitrary A and $\sim A$. (Consider any valuation that relates both A and $\sim A$ to 1, but does not relate B to 1.) Accordingly, we may recognize, without the pain of triviality, sentences (e.g., Liars) that are true and false.

Let $\Vdash_p$ be the consequence relation of *P*, defined as per the classical case (preservation of truth). One feature of $\Vdash_p$ should be noted:

» If $\Gamma \Vdash_p A$ then $\Gamma \Vdash_c A$.

The class of admissible *P*-valuations properly includes the class of classically admissible valuations; hence, if truth is preserved over all admissible *P*-valuations, then it's preserved over the restricted classical class of such valuations. The converse does not hold; for example

$$\sim A, A \vee B \Vdash_c B$$

but

$$\sim A, A \vee B \nVdash_p B$$

That said, it is easy to see that the given converse holds for the *negation-free* fragment. The chief difference between classical logic and *P* concerns negation; and the difference is significant.

Before highlighting some differences between classical- and *P*-negation, the issue of Double Negation should be addressed. One might think that there is something ad hoc about adding Double Negation once Exclusion has been dropped. After all, Double Negation results from the joint work of Exhaustion and Exclusion; and since Exclusion was dropped, so too ought Double Negation be dropped. While I am sympathetic with such thinking, I think that it need not be accepted. Dialetheism, as I advance it, takes the Liar to be true and false — a true sentence A such that $\sim A$ is also true. Such a sentence calls for dropping Exclusion, which significantly affects negation; however, there seems to be no reason — none motivated by the Liar — that directly challenges Double Negation. Until such motivation emerges, I suggest retaining Double Negation.[5]

4.1 *P-theory of negation*

Negation, according to *P*, exhibits many unfamiliar features. Given the motivation behind *P*, the motivation to accommodate apparently true and false sentences, one feature is quite natural and expected, specifically, that $\sim(A \wedge \sim A)$ — the 'law of non-contradiction' — is not logically true. That much, as I said, is expected.[6] What may be startling are other features of negation:

» $\sim A \nVdash_p \sim(A \wedge B)$

» $\sim(A \vee B) \nVdash_p \sim A \wedge \sim B$

» $\sim A \wedge \sim B \nVdash_p \sim(A \vee B)$

» $\sim A \vee \sim B \nVdash_p \sim(A \wedge B)$

While the 'failure' of such familiar principles may be startling, it shouldn't be terribly surprising, at least on reflection. After all, it is precisely Exclusion that, when coupled with the behavior of conjunction and disjunction, ensures the classical behavior of negation. Taking truth as our only primitive semantic ('truth') value, we do not get the familiar De Morgan 'laws' without Exclusion. So, as said, the 'failure' of such 'laws' should not be terribly surprising, at least given the prior rejection of Exclusion.

[5] I should note that dropping Double Negation does not affect the chief 'double-aspect' view that I propose; it simply makes *P*-negation even less constrained than it is with Double Negation. (The extent to which it is 'unconstrained' is discussed below.)

[6] Priest and Sylvan [10] disagree; I discuss some of their remarks in §5.

5 *But is P-negation Negation?*

Surprising or not, the theory of negation reflected in *P* is very unfamiliar, so much so that philosophers will question whether the given negation is 'really' negation. Indeed, Graham Priest and Richard Sylvan [10] have leveled just such objections.[7] Priest and Sylvan launch a number of arguments against the *P*-theory of negation; I will briefly discuss three such arguments.

5.1 *The law of non-contradiction*

The first objection concerns the 'law' of non-contradiction (LNC), at least understood as $\sim(A \land \sim A)$, which is logically true in Priest's *LP*.[8] Priest and Sylvan begin by searching for a reason to reject LNC:

> [P]resumably any case against [LNC] will hinge on the undesirability of secondary contradictions. Conceivably, we might invoke the razor that contradictions should not be multiplied beyond necessity. However, even if this is correct (and is it?) it does not get us very far until we know what 'necessity' is. [10, p 164]

Suppose, as I have, that some A is such that $A \land \sim A$ is true.[9] A 'secondary contradiction' immediately emerges if LNC is accepted,

[7] Priest and Sylvan addressed many of their objections directly against a different paraconsistent logic, namely, DaCosta's C_ω, which is slightly stronger than *P*.

[8] It is worth noting that Priest [8] actually rejects 'another' version of non-contradiction; he rejects $\sim(T\underline{A} \land \sim T\underline{A})$. That Priest rejects the T-ful version of LNC (as it were) is perhaps telling; for in order to reject the T-ful version while accepting the T-free version (as it were), Priest must reject the intersubstitutivity of truth. On the theory that I advance, there is no need to reject the fundamental intersubstitutivity of truth.

[9] Incidentally, Priest and Sylvan call $A \land \sim A$ a 'true contradiction'. That tag is fine, so long as 'contradiction' is understood purely in terms of logical form, namely, as any sentence of the form $A \land \sim A$. Some philosophers use 'contradiction' to mean *an explosive sentence*, a sentence A from which every sentence (logically) follows; in the explosive sense of 'contradiction' no dialetheist thinks that there are true contradictions (except trivialists, who believe that everything is true, but they are non-actual, as far as I know).

namely, $\sim(A \wedge \sim A)$. As Priest and Sylvan note, the 'razor' against such secondary contradictions might be a reason to reject LNC, but that response, as Priest and Sylvan note, requires more work, particularly concerning 'necessity'.

Fortunately, the work on 'necessity' need not be done; the reason to reject LNC has little to do with secondary contradictions. The reason for rejecting LNC is that (so far) we have no reason to accept it, at least given our recognition of sentences that are true and false — sentences that are true and have a true negation. After all, the reason that we imposed Exclusion in our (classical) semantics is that we had no reason to think that some truths had true negations. Then came the surprising Liar, which, coupled with naive truth theory, suggested that some (admittedly peculiar) sentences are both true and false. In turn, our grounds for accepting Exclusion — and, hence, LNC — vanished when we recognized true sentences of the form $A \wedge \sim A$. Accordingly, secondary contradictions have little to do with rejection of LNC.

5.2 *Mere sub-contraries*

Along the vein of LNC Priest and Sylvan launch a further argument:[10]

> Traditionally A and B are sub-contraries if $A \vee B$ is a logical truth. A and B are contradictories if $A \vee B$ is a logical truth *and* $A \wedge B$ is logically false. It is the second condition which therefore distinguishes contradictories from sub-contraries. Now in [*P*] we have that $A \vee \sim A$ is a logical truth. But $A \wedge \sim A$ is not logically false. Thus A and $\sim A$ are sub-contraries, not contradictories. Consequently [*P*-] negation is not negation, since negation is a contradiction forming functor, not a sub-contrary forming functor. [10, p 165]

This argument, like the others, is not strong. Granted, *traditionally* A and B are contradictories iff $A \vee B$ and $\sim(A \wedge B)$ are logically

[10] In the following quote I replace 'Da Costa' by '*P*'.

true. But that is simply the *traditional* theory of negation on which, *nota bene*, $A \wedge {\sim}A$ is *explosive*![11] Appeal to the traditional theory of negation (and, in turn, *contradiction*) does not give reason to accept LNC once Exclusion has been dropped. That negation, according to *P*, is a device that forms only sub-contraries is not in itself an argument against *P*-negation; what is needed is reason to think that, once Exclusion is dropped, we ought still accept the traditional view that negation is a 'contradiction-forming' device.[12]

5.3 Traditional properties of 'real negation'

Setting LNC aside, Priest and Sylvan advance one final argument against *P*-negation. In effect, the argument is that *P*-negation affords too few traditional inferential features, including the ones mentioned in §4.1. Priest and Sylvan argue that such 'failures'

> show that [*P*-] negation has virtually none of the inferential properties traditionally associated with negation.... This is a further piece of evidence suggesting that [*P*-] negation is not really negation. We have now mustered strong evidence to this effect and the case seems pretty conclusive. [10, p 165]

How strong is this argument? Contrary to Priest and Sylvan, the arguments from LNC (and, in turn, sub-contraries) are not strong, and so that part of the 'evidence' needs to be set aside.

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[11] I am not saying that any theory according to which $A \vee {\sim}A$ and ${\sim}(A \wedge {\sim}A)$ are logically true is an explosive theory; that is plainly wrong (as Priest's *LP* shows). The point above is that *traditionally* contradictories are explosive.

[12] Note that I avoid the term 'functor' here. The reason is that if we take truth to be our only primitive semantic value, it is misleading to say that $\sim$ is interpreted as a 'truth function'. Of course, $\sim$ is taken to be a functor — defined by 'truth tables' — so as to avoid the circularity of using 'not' in one's truth conditions for $\sim$-sentences; however, I think it is fairly clear that there is no avoiding such circularity, at least if one thinks that there is no language 'richer' than English in which to give such truth conditions. The relational semantics of §2, which are due essentially to Tarski, does not hide such circularity.

What about the argument from inferential properties? That argument, I think, does not provide reason to accept LNC (or backtrack on the original rejection of Exclusion). After all, the inferential properties at issue (the ones traditionally associated with negation) are simply those that, for the most part, result from imposing Exclusion; and we have been given no reason to 'take back' our initial rejection of Exclusion.

By my lights, then, none of Priest's and Sylvan's arguments provide strong reason to reject the *P*-theory of negation. Is my proposal, then, that the *P*-theory of negation, *on its own*, does adequately characterize negation? Not quite.

5.4 *The double-aspect hypothesis*

The situation, as I see it, is as follows. A dialetheic response to the Liar is a natural and simple move; and with that move it is natural to reject Exclusion. Given that truth is our only primitive 'truth value' a rejection of Exclusion yields *P*. The trouble, as Priest and Sylvan note, is that the *P*-theory of negation yields very unfamiliar inferential properties — very abnormal features.

Given the noted abnormalities, one is tempted to think, as Priest and Sylvan suggest, that *P*-negation — and, generally, any 'negation' that doesn't satisfy the traditional features — just *isn't* negation. But that temptation, I think, ignores an interesting option: that negation has a *double aspect*. The idea is that, on one hand, negation *normally* behaves according to Exclusion, normally exhibits 'classical behavior'; on the other hand, when it is involved in a paradoxical construction, negation exhibits freedom from Exclusion — it is in many ways 'free-floating', as witnessed in *P*. To be sure, its 'free-floating' behavior still strikes us as odd, but that is simply because such behavior is *rare* and *abnormal*. Indeed, the motivation for rejecting Exclusion came from a very abnormal source — paradoxical sentences that, except for grammatical necessity (or, perhaps, the odd contingent twist of affairs), play little role in our normal inferential practices involving negation. And it is important to recall that our 'intuitions' about 'normal negation-behavior' are formed by familiar cases; they are not formed by the odd Liar-like sentences.[13]

My suggestion, then, is that negation enjoys a double-aspect. Normally, negation behaves classically; however, when it is involved in 'paradoxical set-ups' negation exhibits the 'free-floating' behavior reflected in *P*. The task of the next section is to make the double-aspect theory precise.

6 Double-Aspect Negation: The Logic AP

The double-aspect theory of negation may be formalized via a non-monotonic logic, specifically, what Diderik Batens [1] calls an *adaptive logic*. I dub the target logic '*AP*' (for *aspectival P*).[14] I will present the core ideas, and then the target philosophical interpretation.

Aspectival Negation: AP

Let $\mathcal{R}$ be any admissible *P*-valuation (as per §4) and p any atomic sentence (propositional parameter). Define $\mathcal{R}^\star$, an *inconsistency measure of* $\mathcal{R}$, as follows:

» $\mathcal{R}^\star = \{p : \mathcal{R} \models p \text{ and } \mathcal{R} \models {\sim}p\}$

Next, define the following relation on admissible *P*-valuations:

» $\mathcal{R}_i \prec \mathcal{R}_j$ iff $\mathcal{R}_i^\star \subset \mathcal{R}_j^\star$

[13] This is in large part why the paradoxical sentences are so surprising upon discovery: they show us exceptions to the norm, as it were.

[14] I learned of Batens' work after formulating *AP*. As far as I can see, the logic that I dub '*P*' is slightly stronger than what Batens calls '*CLuN*', and my target logic, namely *AP*, is slightly stronger than what Batens calls '*ACLuN2*'. While I do not want to needlessly proliferate names, I will none the less stick to my original tags.

We say that $\mathcal{R}_i$ is *less inconsistent than* $\mathcal{R}_j$ iff $\mathcal{R}_i \prec \mathcal{R}_j$. In turn, define a *minimally inconsistent model* (MI-model) thus:

» $\mathcal{R} \vDash_{mi} \Gamma$ iff $\mathcal{R} \vDash \Gamma$ and if $\mathcal{R}_i \prec \mathcal{R}$ then $\mathcal{R}_i \nvDash \Gamma$

Intuitively, $\mathcal{R}$ is an MI-model of Γ just if it is a model of Γ and any (admissible) valuation less inconsistent than $\mathcal{R}$ fails to be a model of Γ. The idea, in effect, is that MI-models 'seek' the least inconsistent way to model (satisfy) a set of sentences; in particular, any classical model of Γ will be an MI-model of Γ. (Recall that admissible *P*-valuations include the class of classically admissible valuations.)

The logic *AP* results from defining consequence over minimally inconsistent models:

» $\Gamma \Vdash_{ap} A$ iff any MI-model of Γ is a model of A.

Logical truth (valid sentence) may be defined as truth in every (admissible) valuation: $\mathcal{R} \vDash A$.[15]

6.1 Non-monotonicity

That *AP* is non-monotonic is plain:

$$\sim p, p \vee q \Vdash_{ap} q$$

but

$$p, \sim p, p \vee q \nVdash_{ap} q$$

What is important to note is that the consequences of Γ in *AP* are precisely the 'standard', classical consequences of Γ *if* Γ is consistent; otherwise, the consequences reflect the free-floating *P*-behavior of negation, which results only when p is both true and false.

[15] Suppose that one defines *logical truth* thus: A is logically true in *AP* iff $\emptyset \Vdash_{ap} A$. That does not work. On that account, A may be 'logically true' without being true in every admissible valuation. Consider $\sim(p \wedge \sim p)$, which is not true in every admissible $\mathcal{R}$, but it does follow from $\emptyset$ in *AP*. (Every admissible $\mathcal{R}$ is a model of $\emptyset$, and any classical model is an MI-model. So, that $\emptyset \Vdash_{ap} \sim(A \wedge \sim A)$ is not surprising but, as said, it is insufficient for logical truth.)

6.2 *The philosophical import*

The philosophical import of *AP* is the double-aspect hypothesis: that negation is aspectival. Recall the lesson of the Liar. According to dialetheists, the lesson is simply that some sentences are true and false. That, by my lights, is correct, as far as it goes; however, that lesson is incomplete. The lesson is not only that there are sentences that are true and false; the lesson is that *negation* is aspectival. *AP* is intended to record the aspectival natural of negation.

The idea is that *except* for the odd paradoxical sentences — sentences that arise out of mere grammatical necessity (or the rare twist of contingent affairs) — negation behaves precisely as philosophers have always taken it to behave, namely, classically. When it is involved (by grammatical necessity or the rare twist of contingent affairs) in a paradoxical construction — a sentence that, due to the overall workings of the language, has no way of being true without thereby also being false — negation exhibits free-floating behavior. Truth remains our only primitive semantic value; and falsity remains a derivative notion, defined as always in terms of (aspectival) negation and truth.

What is attractive about the double-aspect hypothesis is that it respects many 'intuitions' about negation. Consider, for example, the standard intuition that LNC is a logical truth. That intuition, like most classical intuitions, is founded on the *normal* behavior of negation. The Liar, of course, is abnormal; it is linguistic residue of grammar. But when the Liar is taken seriously, one soon finds — on pain of expressive difficulties — that the Liar challenges our classical intuitions. The double-aspect hypothesis is that those classical intuitions needn't be rejected so much as slightly modified: one needs to recognize that negation enjoys a double life, as it were. LNC (contrary to Priest and Sylvan) is not valid simpliciter; however, it is valid over the restricted class of sentences on which our classical intuitions are built. The same considerations apply to all classical intuitions about negation: they are correct over the vast range of (non-paradoxical) sentences on which they were formed. But the vast range of sentences on which our classical intuitions were formed does not exhaust the entire range of sentences. When paradoxical constructions are at hand, negation behaves oddly; it exhibits behavior that we rightly think to be strange — it *is* strange, at least in the sense of being abnormal, deviating from the normal sentences on which our intuitions are built.

Another virtue of the double-aspect hypothesis is that it explains why philosophers cringe — or wield an incredulous stare — at the dialetheic response to paradox. Many philosophers simply declare (sometimes with a fist-thump) that no sentence can be both true and false; they simply declare that *the very meaning of falsity* rules out dialetheism. The double-aspect theory affords a nice explanation of such declarations. After all, 'is false' is *normally* associated with the *normal* behavior of negation! And in that *normal* aspect of negation the given declaration is perfectly correct: no sentence can be true and false, where 'is false' is restricted to truth of negation *as negation normally behaves*. Once the two aspects of negation are distinguished, dialetheists may (and should) join in the given declaration.

In the end, then, *AP* is intended to reflect the aspectival nature of negation (and, derivatively, falsity). We retain a single, primitive 'true value' while, for reasons having to do with paradox, rejecting Exlcusion. But the rejection of Exclusion, according to the double-aspect hypothesis, is not so much a rejection of our 'exclusive intuitions' as it is a modification of our negation-theory. Exclusion still holds over the vast range of sentences with which we normally reason; it fails only for odd constructions that cry out for a free-floating negation — and, hence, a free-floating notion of falsity. *AP* is intended to reflect just such play between 'normal' and 'free-floating' falsity.

7 *Further Virtues of Double-Aspect Dialetheism*

I have already mentioned some of the explanatory virtues of the double-aspect hypothesis, particularly concerning classical intuitions about negation and falsity. My aim in this section is to (very briefly) indicate a few advantages that double-aspect dialetheism has over its 'orthodox' rival — where its orthodox rival is Priest's *LP*-based version.

7.1 *Field's Criticism*

Hartry Field [5] recognizes the immediate advantage that a dialetheic response to paradox affords, namely, retaining the fundamental intersubstitutivity of truth. Let T and F be our truth and falsity predicates. Given the intersubstitutivity of truth and the mere derivative 'nature' of falsity, $\sim\mathsf{T}\underline{A}$ is equivalent to $\mathsf{T}\underline{\sim A}$, which is equivalent to $\mathsf{F}\underline{A}$.

Dialetheism is the view that some sentences are true and false. That is the way 'dialetheism' has usually been defined; and that is precisely the view that dialetheists hold. Field's criticism is that, given the intersubstitutivity of truth (and the resulting equivalences with falsity), dialetheists are stuck in a very odd situation. Field advances two criticisms, both aimed at oddities involved with typical ways of characterizing dialetheism. The first criticism runs thus:

> [A] problem with defining dialetheism as the doctrine (D) *that certain sentences are both true and false* is that while a dialetheist should certainly assert
>
> i) $\mathsf{T}\underline{A} \wedge \mathsf{F}\underline{A}$
>
> for certain A (e.g. the Liar sentence), he should deny this [i.e., assert the negation of it] as well. For the dialetheist asserts both $\mathsf{T}\underline{A}$ and $\mathsf{F}\underline{A}$. But from $\mathsf{F}\underline{A}$ we get [via the noted equivalences] $\sim\mathsf{T}\underline{A}$ ... ; so
>
> ii) $\mathsf{F}\underline{A} \wedge \sim\mathsf{T}\underline{A}$

which surely entails the negation of (i).

> ...Of course, it is a consequence of dialetheism that some sentences are both true and false, and there's no particular problem in the fact that the particular sentence (D) is among them. But what is odd is to take as the doctrine that defines dialetheism something that the dialetheist holds to be false as well as true. [5]

Does (ii) 'surely entail' the negation of (i)? On the orthodox (single-aspect) version of dialetheism, underwritten by *LP*,[16] the answer is *yes*. On the double-aspect view, underwritten by *AP*, the answer is *no*.

Field's faulty presupposition, then, is that negation has a single aspect. The standard definition of 'dialetheism', Field's (D), is unproblematic given the double-aspect character of negation and, in turn, falsity. Indeed, it is precisely (and only) when some A is both true and false that the given entailment — which involves normal De Morgan properties — fails. A double-aspect dialetheist, unlike Priest, need not hold that her definitive doctrine is false.

Field's second criticism is the observation that

> it is misleading to characterize the dialetheist's attitude towards, say, the Liar sentence as the view (i) *that it is both true and false*, when one could equally well have characterized it as the view (iv) *that it is neither true nor false*, or as the view (ii) *that it is false and not true*, or the view (iii) *that it is true and not false*. [5]

Once again, Field is presupposing that negation behaves normally; for the various listed ways of characterizing the dialetheist's attitude are 'equally good' only if normal De Morgan principles hold. Targeted, as it is, against orthodox dialetheism Field's observation is important; however, the observation is off the mark against double-aspect dialetheism.

[16] or, for that matter, underwritten by Priest's own adaptive logic 'minimally inconsistent *LP*' [9]

7.2 *Shapiro's Challenge*

Dialetheism, as I mentioned, is a very attractive response to paradox because (among other things) it avoids the expressive problems that perennially confront its rivals. Stewart Shapiro [12] challenges that (alleged) virtue of dialetheism.

Shapiro agrees that dialetheism avoids the usual expressive problems confronted by non-dialetheic rivals; he agrees that it does not need to invoke an 'essentially richer meta-language' or forbid 'quantification over contexts' or so on. (After all, the motivation for such familiar restrictions is to avoid having to recognize that some sentences are true and false.) But while dialetheism avoids such typical problems, Shapiro contends that it confronts at least an analogous expressibility problem; specifically, the dialetheist has no way of expressing the apparently important notion of a ***non-dialetheia*** — a sentence that is not both true and false.

Shapiro's paper is rich in its discussion and I will not attempt to address all of his arguments here; indeed, I will simply isolate one of his arguments and indicate how double-aspect dialetheism avoids the criticism.[17] Shapiro's challenge runs thus:[18]

> There are a number of (perhaps non-equivalent) ways to indicate that a given sentence A is a dialetheia. In the 'object language' (so to speak), one can just assert $A \wedge {\sim}A$. Or one can say that A is true and false ($T\underline{A} \wedge T\underline{\sim A}$) or that A is true and not true ($T\underline{A} \wedge {\sim}T\underline{A}$). But how can one say that A is a non-dialetheia? It will not do to simply say ${\sim}(A \wedge {\sim}A)$. For this last is a logical truth in Priest's semantics [i.e., 'orthodox dialetheism']. It holds

[17] Of course, Shapiro's criticisms, like Field's, are directed at orthodox dialetheism. Like Field, he recognizes the apparent virtues of dialetheism but finds its orthodox version wanting. Part of my aim in this paper is to put double-aspect dialetheism on the table, especially since, by my lights, it is a much more natural version of dialetheism, one that respects and explains 'classical intuitions' while affording the fruits of dialetheism, in general.

[18] I change Shapiro's symbolism for uniformity's sake. Nothing hinges on the change.

> no matter what sentence A is. Priest points out in several places that if A is a dialetheia, in the sense that $A \wedge \sim A$ is true, then $\sim(A \wedge \sim A)$ is another dialetheia. That is, we have both $A \wedge \sim A$ and $\sim(A \wedge \sim A)$. So if 'A is a non-dialetheia' is defined as '$\sim(A \wedge \sim A)$', then every sentence is a non-dialetheia, including every dialetheia. [12]

That Shapiro's challenge, at least as formulated above, does not apply to double-aspect dialetheism is plain. As Shapiro makes clear, the problem with defining 'A is a non-dialetheia' as the falsity of $A \wedge \sim A$, and so the truth of $\sim(A \wedge \sim A)$, is that every sentence is thereby a non-dialetheia, including every dialetheia. But that problem (as Shapiro goes on to note) is not a problem if LNC is invalid, and it is invalid in *AP*.

There is more to Shapiro's challenge. Let A be a dialetheia. Then, by definition, $A \wedge \sim A$ is true. But, at least given *AP*, it is 'logically possible' that $\sim(A \wedge \sim A)$ is also true — at least where 'logical possibility' is defined in terms of *AP*-models. So, some dialetheia might also be a non-dialetheia, and hence the given definition of 'non-dialetheia' is not exclusive, in the sense that A might be both a dialetheia and a non-dialetheia.[19]

Is the non-exclusivity of 'non-dialetheia' a problem? I think that it is not a problem, at least for the double-aspect view. After all, the 'non' in 'non-dialetheia' will itself be aspectival, in as much as it is negation, which is aspectival. Accordingly, if (as the grammar

[19] Note that there is an *AP* 'trivial model', the model in which every sentence is true (and, hence, also false). That feature is shared with *LP* (and, more generally, *FDE*). I have some reservations about that, especially since the only motivation for recognizing 'gluts' arises from a 'small' class of very peculiar, abnormal sentences. But I will leave that issue for a larger project. (One route I've explored is to divide the atomic sentences into disjoint classes, and then define admissible valuations in such a way that only one of the two classes can be 'glutty'. Intuitively, the idea is that our target language — the language modeled by the formal language — already affords such a distinction (e.g., paradoxical sentences and otherwise), even though there is no decidable method for distinguishing the classes. But, again, I leave that issue for a larger project.)

always seems to ensure) there are some constructions in which 'non' in 'non-dialetheia' forces abnormal (paradoxical) behavior, then the overall intersection of 'is a dialetheia' and 'is a non-dialetheia' will be non-empty.[20] That said, the double-aspect hypothesis does not (as far as I can see) *force* the claim that 'non-dialetheia' is actually non-exclusive, that there are dialetheia that are also non-dialetheia. To be sure, in as much as *AP*-models are taken to represent 'logical possibility', there is (thereby) the logical possibility of such non-exclusivity; however, as far as I can see, there is no need to take the given models in that way.[21]

For present purposes, the important point is that double-aspect dialetheism, unlike its orthodox single-aspect version (represented in *LP* or its adaptive counterpart), seems to avoid Shapiro's challenge. Indeed, given *AP*, one can successfully express (perhaps, at times, with the help of pragmatics) Shapiro's target claims, and do so in the natural way that Shapiro suggests:

» *that* A *is true only* may be expressed as $A \wedge {\sim}(A \wedge {\sim}A)$.

» *that* A *is false only* may be expressed as ${\sim}A \wedge {\sim}(A \wedge {\sim}A)$.

Pragmatics, as mentioned, may be needed at times, but that is not a problem; pragmatics will inevitably be invoked in any final analysis of English. As Shapiro argues, the situation with respect to 'orthodox' dialetheism appears (at least prima facie) to be different: pragmatics will not help, given the validity of LNC in *LP*. At least on that score, double-aspect dialetheism is preferable.[22]

[20]Note the 'non' in 'non-empty', and the implicit 'not' in 'exclusive'! In normal cases, all such 'not's are unravelled classically, but the double-aspect view recognizes that, due to grammatical residue, there may be abnormal, free-floating behavior too.

[21]Few philosophers will say that it is logically possible that grass is red but not colored, despite the entrenchment of Tarskian (classical) logic. (There are classical Tarskian models in which grass is red but not colored, at least on the standard 'translation' or regimentation of English into classical logic.)

[22]Joachim Bromand [4] raises what, in effect, is the same expressibility challenge

8 Closing Remarks and Further Directions

In this paper I have advanced what I call a double-aspect theory of negation, and in particular the theory of negation reflected in *AP*. While I have not argued the point (but, instead, relied on other literature), dialetheism appears to be a natural lesson to draw from the Liar paradox (and its kin). One obstacle to dialetheism has always been the 'gut-feeling' that the very meaning of *falsity* rules out the dialetheic lesson. That reaction, I have suggested, ignores the hypothesis that negation is ultimately aspectival, that negation generally exhibits classical behavior but, given odd (grammatically necessitated) sentences, sometimes behaves non-classically — indeed, behaves in a very free-floating fashion.

The double-aspect hypothesis respects the strong, classically-minded intuitions that many philosophers have about negation; it respects such intuitions by *preserving* them — at least with respect to the normal, non-paradoxical cases, which are precisely the ones on which such intuitions are founded. By my lights, the advantages of a dialetheic response to paradox — in particular, the preservation of expressive appearances — are strong; and the virtue of double-aspect dialetheism is that it retains such advantages while none the less retaining the insights of classical logic. Moreover, as indicated (albeit briefly) in §7, double-aspect dialetheism, unlike its orthodox (single-aspect) counterpart, seems to avoid many of the oddities that have troubled those who have taken dialetheism seriously.

My aim in this paper has been to introduce the double-aspect view of negation (or double-aspect dialetheism). My hope is that the aim has been achieved and that the view is interesting and promising enough to foster further consideration. Before closing, however, I

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that Shapiro raises. Bromand argues that orthodox dialetheism, underwritten by *LP*, cannot express that A is a non-dialetheia. As should be clear, double-aspect dialetheism does not fall prey to Bromand's version of the objection, which turns on the 3-valued semantics of *LP*.

will mention an issue on which I have said very little: *conditionals*.

What (if any) conditional is at play in *AP*? At the very least, there is a 'material-like' conditional.[23] Suppose, for example, that $A \rightarrow B$ is defined in the 'original' classical semantics: $\mathcal{R} \Vdash A \rightarrow B$ iff either $\mathcal{R} \Vdash {\sim}A$ or $\mathcal{R} \Vdash B$. In the classical semantics, $\rightarrow$ will be just the regular (classical) hook — the material conditional. But what happens when, after confronting the Liar, Exclusion is dropped? As discussed, negation becomes 'free-floating', and so $\rightarrow$, being defined in terms of $\sim$, will itself exhibit abnormal behavior. $A \rightarrow A$ remains valid, but abnormality will none the less abound. For example, modus ponens will fail, as will many other traditional inferential properties; however, such abnormalities of $\rightarrow$ piggy-back on the double-aspect of $\sim$, the upshot being that $\rightarrow$ will behave classically for the vast range of sentences on which our traditional intuitions about 'if' are grounded.[24]

I close by clearing away one worry. One might think that the failure of modus ponens immediately undermines the entire project, since, one might think, with the failure of modus ponens one no longer derives the Liar paradox, and hence no longer derives that some (admittedly peculiar) sentence is true and false, and hence that there is no reason to think that negation has a double-aspect. That would be a devastating objection were it correct; however, it is not correct. What is interesting (and in some ways quite astonishing) is the sheer persistence of the Liar; the paradox persists even in the very weak logic *P*. Recall that $A \rightarrow A$ and, in turn, $A \leftrightarrow A$ remain valid in *AP*, at least on the given 'material-like' definition. Given

[23] Some do not take the 'material conditional' to be a genuine conditional. In other work, I hope to bolster *AP* with intensionality, perhaps thereby alleviating some of the concerns about the material conditional.

[24] Strengthening $\rightarrow$ by adding modality to *AP* should yield similar results. I leave that project for another venue. (If, contrary to my current thinking, an aspectival approach to the conditional proves not to be viable, there are suitable conditionals to do the trick. See [8].)

intersubstitutivity of $\mathsf{T}\langle A\rangle$ and A, we have $\mathsf{T}\langle A\rangle \leftrightarrow A$. As above, $A \rightarrow B$, by definition, is $\sim A \vee B$, in which case $\mathsf{T}\langle A\rangle \leftrightarrow A$ is

$$(\sim\mathsf{T}\langle A\rangle \vee A) \wedge (\sim A \vee \mathsf{T}\langle A\rangle)$$

Consider the Liar λ, which is $\sim\mathsf{T}\langle\lambda\rangle$. The λ-instance of $\mathsf{T}\langle A\rangle \leftrightarrow A$ is $\mathsf{T}\langle\lambda\rangle \leftrightarrow \sim\mathsf{T}\langle\lambda\rangle$, which by definition is

$$(\sim\mathsf{T}\langle\lambda\rangle \vee \sim\mathsf{T}\langle\lambda\rangle) \wedge (\sim\sim\mathsf{T}\langle\lambda\rangle \vee \mathsf{T}\langle\lambda\rangle)$$

Even in *P* (and, hence, in *AP*), the left conjunct yields $\sim\mathsf{T}\langle\lambda\rangle$ while the right conjunct yields $\mathsf{T}\langle\lambda\rangle$. Hence, even in *P*, and so without modus ponens, the original paradox arises: λ is a sentence that is both true and false; it is a sentence in which negation is on holiday.[25]

References

[1] Diderik Batens. "A Survey of Inconsistency-Adaptive Logics". In D. Batens, C. Mortensen, G. Priest, and J-P Van Bendegem, editors, *Frontiers of Paraconsistency*, pages 49–73. Research Studies Press, King's College Publications, Baldock, 2000.

[2] JC Beall. "Simply Semantic Inconsistency". to appear, 2003.

[3] JC Beall and Bas C. van Fraassen. *Possibilities and Paradox: An Introduction to Modal and Many-Valued Logic*. Oxford University Press, Oxford, 2003.

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[4] Joachim Bromand. "Why Paraconsistent Logic Can Only Tell Half the Truth". *Mind*, 111(444):741–749, 2002.

[5] Hartry Field. "Is the Liar Both True and False?". In JC Beall and Brad Armour-Garb, editors, *Deflationism and Paradox*. Oxford University Press, Oxford, 2003. Forthcoming.

[6] Robert L. Martin, editor. *Recent Essays on Truth and the Liar Paradox*. Oxford University Press, New York, 1984.

[7] Vann McGee. *Truth, Vagueness, and Paradox*. Hackett, Indianapolis, 1991.

[8] Graham Priest. *In Contradiction: A Study of the Transconsistent*. Martinus Nijhoff, The Hague, 1987.

[9] Graham Priest. "Minimally Inconsistent LP". *Studia Logica*, 50:321–331, 1991.

[10] Graham Priest and Richard Routley. "Systems of Paraconsistent Logic". In Graham Priest, Richard Routley, and Jean Norman, editors, *Paraconsistent Logic: Essays on the Inconsistent*, pages 151–186. Philosophia Verlag, 1989.

[11] R. M. Sainsbury. *Paradoxes*. Cambridge University Press, Cambridge, Second edition, 1995.

[12] Stewart Shapiro. "Simple Truth, Contradiction, and Consistency". In Graham Priest et al., editor, *New Essays on (Non-) Contradiction*. Oxford University Press, 2003. forthcoming.

[13] Keith Simmons. *Universality and The Liar*. Cambridge University Press, 1993.