Two flavors of Curry's paradox

Jc Beall & Julien Murzi*

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Abstract

In this paper, we distinguish two versions of Curry's paradox: *c-Curry*, the standard *conditional*-Curry paradox, and *v-Curry*, a *validity*-involving version of Curry's paradox that isn't automatically solved by solving c-curry. A unified treatment of Curry's paradox thus calls for a unified treatment of both c-Curry and v-Curry. If, as is often thought, c-Curry paradox is to be solved via non-classical logic, then v-Curry may require a lesson about the structure—indeed, the *sub*structure—of the validity relation itself.

It is generally agreed that one of the hardest among the paradoxes is Curry's paradox.[1] Many have thought that the notorious liar paradox may be resolved by adjusting our theory of (the rules governing) negation. Perhaps, as on common *paracomplete* options, negation fails to be *exhaustive*: it fails to classify sentences as

*Universities of Connecticut and Otago [jc.beall@uconn.edu] & University of Kent and Munich Center for Mathematical Philosophy, Ludwig-Maximilians Universität [j.murzi@gmail.com]. This paper came about after the happy discovery that we (authors) had independently stumbled on the same v-Curry paradox, the distinction between it and its original c-Curry version, and the philosophical upshot that it appears to have for some currently much-discussed approaches to paradox (what we call rcf approaches here). For valuable discussion along the way we thank Phillip Bricker, Colin Caret, Roy Cook, Aaron Cotnoir, Hartry Field, Luca Incurvati, Jeffrey Ketland, Hannes Leitgeb, Graham Priest, Agustín Rayo, Stephen Read, Greg Restall, David Ripley, Lionel Shapiro, Bruno Whittle, and Crispin Wright. Thanks too to the UConn Logic Group, and to participants of a reading group on validity and truth preservation at St Andrews, and at various related events at the University of St Andrews and, more recently, the NIP centre at the University of Aberdeen, and also at the University of Minnesota, and at the University of Melbourne, and various AAL and AAP meetings. Murzi warmly thanks the Analysis Trust and the Alexander von Humboldt Foundation for, respectively, doctoral and post-doctoral research funding, which supported work on this paper. Beall warmly thanks the Holbox community for a delightful setting in which to discuss an early version of this paper.

[1]See Curry (1942), Geach (1955), Prior (1955), Myhill (1975), Meyer et al. (1979), Beall et al. (2006), Priest (2006), Beall (2007b), Field (2008) and Beall (2009). We review the so-called truth-theoretic version of the paradox in §1. Our points below carry over to the 'set-theoretic' or *exemplification* version. See Beall (2008) for very general background.

being either true or not true, thus allowing for 'gaps' between truth and falsity.[2] Perhaps, as on common *paraconsistent* options, negation fails to be *exclusive*: it allows for sentences to be 'glutty', or both true and false.[3] But while the liar paradox may be blocked via a non-classical theory of negation, Curry's paradox arises even in negation-free languages, and in particular in those theories that enjoy unrestricted fundamental semantic principles for truth (e.g., T-biconditionals) or exemplification (e.g., naïve comprehension). The main challenge for such theories is Curry's paradox.

In this paper, we focus our attention on currently much-discussed *robustly contraction-free* ('rcf' for short) theories.[4] All such theories attempt to resolve Curry's paradox by keeping the naïve principles for truth and exemplification, on one hand, and by rejecting the existence of certain kinds of connectives, on the other—*contracting connectives*, as we explain in §3. Our aim in this paper is not to question the viability or promise of such theories in general; we assume their viability throughout. Our chief aim is to show that there is more to Curry paradox than its standard (conditional-involving) version, and that rejecting contracting *connectives* is prima facie insufficient for solving curry paradox in general.

We distinguish two versions of Curry's paradox: *c-Curry*, the standard *conditional*-involving version (which is usually dubbed *Curry's paradox*), and *v-Curry*, a *validity*-involving version of Curry's paradox that isn't automatically solved by solving c-Curry. One can think of the difference as a difference in targets: c-Curry is often taken as telling us something about the 'operational rules' governing connectives (e.g., rules governing conditionals); v-Curry tells us something about the 'structural rules' governing the validity or consequence relation itself.[5]

The paper is structured as follows. §1 reviews c-Curry paradox. §2, in turn, reviews the target diagnosis on which we focus—namely, maintaining a detachable (*modus ponens*-satisfying) conditional but giving up a deduction-theorem link between it and validity. §3 briefly rehearses a notably odd, though recently much-discussed, feature of robustly contraction-free theories, namely, that they must, on

[2]See e.g. Kripke (1975), Martin and Woodruff (1975), Maudlin (2004), and Field (2008), and early precedents in Martin (1970). (This is not to say that any such theories contain the claim that paradoxical sentences *are* 'gaps'. We use the notion suggestively, in keeping with terminology in the literature.)

[3]See e.g. Priest (1979, 2006) and Beall (2009), and also Asenjo (1966), Dowden (1984), and Woodruff (1984).

[4]The terminology is from Restall (1993). Recent representative theories in question include those of Brady (2006), Priest (2006), Field (2003), Field (2007), Field (2008) and Beall (2009).

[5]The distinction between operational rules, i.e. rules that essentially involve logical *operators*, and structural rules, i.e. rules that don't, is highly context-sensitive, being relative to the way logic is formalized. We use the distinction suggestively, and we do not consider it to be essential to the distinction between v-Curry and c-Curry.

pain of triviality, lack the claim that valid arguments are truth-preserving (at least on natural, non-vacuous ways of understanding that claim). Towards highlighting what we take to be a more fundamental issue concerning validity, §4 presents v-Curry paradox, and §5 discusses a few of its apparent consequences. §6, in turn, discusses a few avenues of reply to v-Curry, concentrating mostly on what is prima facie the most natural reply for target non-classical theories. §7 offers some concluding remarks.

1 Standard recipe: c-Curry paradox

The standard version of Curry's paradox, what we're calling *c-Curry*, involves a conditional that says of itself (only) that if it's true then everything is true (or some such absurd consequent). There are a variety of well-known versions of c-Curry; we concentrate on what is the simplest for purposes of comparison with our target paradox, namely, v-Curry. In particular, we focus on a version of c-Curry that employs Conditional Proof.[6]

Assume that our truth-predicate unrestrictedly satisfies the *T-Schema*:

$$(\textit{T-Schema}) \vdash Tr(\ulcorner \alpha \urcorner) \leftrightarrow \alpha$$

By some means or other of achieving self-reference (e.g., diagonalization, quotation, etc.), we get a sentence γ which is equivalent to one saying that if γ is true, then everything is true (or some such absurd consequent):[7]

$$\gamma \leftrightarrow (Tr(\ulcorner \gamma \urcorner) \rightarrow \bot)$$

We may then reason as follows (here dropping the truth predicate for simplicity):[8]

[6]We should note that our chief concern, namely, the structural similarity of c-Curry and what we call v-Curry remains for any of the standard versions of c-Curry. But, given the familiarity of Conditional Proof, the conditional-proof version affords the simplest and most efficient presentation.

[7] Perhaps the most intuitive way to think about how such a sentence might emerge is to think about having a name b denoting the sentence $Tr(b) \rightarrow \bot$, so that the T-biconditionals, to which we appeal below, yield $Tr(b) \leftrightarrow (Tr(b) \rightarrow \bot)$.

[8]An alternative version appealing to the so-called rule of *Contraction*

$$(\textit{Contraction})\ \alpha \rightarrow (\alpha \rightarrow \beta) \vdash \alpha \rightarrow \beta$$

1.	$\gamma \leftrightarrow (\gamma \to \bot)$	[T-biconditionals]
2.	$\mid \gamma$	[Assume, for Conditional Proof]
3.	$\mid \gamma \to \bot$	[1, 2; MP]
4.	$\mid \bot$	[2, 3; MP]
5.	$\gamma \to \bot$	[2–4; Conditional Proof]
6.	γ	[1, 5; MP]
7.	$\bot$	[5, 6; MP]

Clearly, MP and Conditional Proof are the main operational rules at work. There is, however, also a deeper, structural rule governing the consequence relation itself, namely, *Structural Contraction*:

(*Structural Contraction*) If $\Gamma, \alpha, \alpha \vdash \beta$ then $\Gamma, \alpha \vdash \beta$.[9]

The validity of this rule is here presupposed in the subderivation, where γ gets used twice, and both uses are discharged by just one application of Conditional Proof at line 5.[10] Without it, one couldn't legitimately apply Conditional Proof, and c-Curry would be blocked.

Rejecting Structural Contraction, though, is not the strategy pursued by standard rcf theories. These theories keep Structural Contraction, and seek to block c-Curry by weakening the operational rules for the conditional. Presumably, the rationale behind this choice is that structural rules are assumed to be more basic, and hence more difficult to abandon. For instance, Hartry Field, a leading rcf-theorist, takes the revision of substructural rules to be 'radical', and suggests that, in any event, it is not needed:

> I haven't seen sufficient reason to explore this kind of approach (which I find very hard to get my head around), since I believe we can do quite

is important for some of the target theories:

1.	$\gamma \leftrightarrow (\gamma \to \bot)$	[T-biconditionals]
2.	$\gamma \to (\gamma \to \bot)$	[1; Simplification]
3.	$\gamma \to \bot$	[2; Contraction]
4.	γ	[1, 3; MP]
5.	$\bot$	[3, 4; MP]

We will return to the rule of Contraction in §2 and §6 below.

[9]This rule is to be sharply distinguished from the *operational* rule of Contraction introduced in footnote 8. The former explicitly (and only) concerns an operator (or connective); the latter concerns the validity relation (e.g., the turnstile) itself.

[10]Multiple discharge discharge of assumptions in a natural deduction framework is in effect equivalent to Structural Contraction. See Negri and von Plato (2001, Ch. 8).

well without it. ... I will take the standard structural rules for granted. (Field, 2008, pp. 10-11)

We return to Structural Contraction in §6, after presenting, in §3, what is prima facie a sufficient reason to explore the substructural approach (viz., v-Curry). For now, we focus on target rcf theories that attempt to resolve Curry's paradox by retaining structural contraction (and other structural rules).

2 Diagnosis

If Structural Contraction is retained, theories enjoying all instances of the T-Schema (and the resources to yield c-Curry sentences) need to reject one of the two highlighted operational rules involved in the c-Curry derivation—in particular, one of the rules for the T-*conditional* (i.e., the conditional involved in the T-Schema).

Consider, first, the operational rule of Conditional Proof. Giving it up requires giving up the strong deduction-theorem link with validity that is often associated with conditionals, namely,

$$\text{(VC)}\ \alpha \vdash \beta \ \text{ iff } \ \vdash \alpha \to \beta.$$

And indeed, what c-Curry is often taken to show is that a deduction-theorem link between validity and one's conditional is the price of having a *detachable* conditional (for use in the T-biconditionals). The point can be made via another version of c-Curry, one that turns on what is sometimes called *Pseudo Modus Ponens*:[11]

$$\text{(PMP)}\ \vdash \alpha \wedge (\alpha \to \beta) \to \beta.$$

This principle immediately yields c-Curry-driven triviality as follows:

1.	$\gamma \leftrightarrow (\gamma \to \bot)$	[T-biconditionals]
2.	$\gamma \wedge (\gamma \to \bot) \to \bot$	[PMP]
3.	$\gamma \wedge \gamma \to \bot$	[1,2; substitution of equivalents, viz. $\gamma \to \bot$ and γ]
4.	$\gamma \to \bot$	[3; substitution of equivalents, viz. $\gamma \wedge \gamma$ and γ]
5.	γ	[1, 4; MP]
6.	$\bot$	[4, 5; MP]

[11] This terminology, as far as we can tell, was first aired in Priest (1980) and later used in Restall (1993) and Restall (1994), and subsequently picked up by others. Notation: throughout, we let $\wedge$ bind more tightly than $\to$, so that $\alpha \wedge \beta \to \gamma$ is equivalent to $(\alpha \wedge \beta) \to \gamma$.

So, if the T-conditional is detachable, that is, satisfies MP

$$(\text{MP})\ \alpha \wedge (\alpha \rightarrow \beta) \vdash \beta,$$

then PMP needs to be invalid (assuming, as we shall throughout, that substitution of equivalents is in effect, similarly for features of conjunction). But, then, one cannot have the deduction-theorem link—which is to say that Conditional Proof is gone too.

One might, of course, instead take c-Curry to show that logic should be devoid of a detachable (i.e., MP-satisfying) connective. But this has not generally been seen as a plausible route, and we say nothing more about it here.[12]

Our focus here is on an increasingly popular route among recent non-classical theorists: namely, the route of 'robust contraction freedom', which involves rejecting the existence of any *contracting connective*. Let a binary connective $\odot$ be *contracting* just if, where $\rightarrow$ is a detachable T-conditional, the conditions C1–C3 hold:

C1. $\alpha \rightarrow \beta \vdash \alpha \odot \beta$;

C2. $\alpha, \alpha \odot \beta \vdash \beta$;

C3. $\alpha \odot (\alpha \odot \beta) \vdash \alpha \odot \beta$.

Then, as we've already anticipated, a theory is *robustly contraction-free* just if it lacks a contracting connective (Restall, 1993).

Any contracting connective gives rise to a c-Curry paradox.[13] We give one example, following the Conditional Proof approach discussed in §1, and so assume one more condition, namely, the analogue of Conditional Proof for $\odot$, what we might call the rule of '$\odot$ Proof',

C4. If $\alpha \vdash \beta$ then $\vdash \alpha \odot \beta$.

And now a $\odot$ version of c-Curry paradox follows the now-familiar pattern. In

[12] At least one of us (Beall, 2011) has been rethinking this route, pursuing a program in which we have all semantic predicates in play but no detachable *connective*. This program has at least one notable and major attractive feature, viz., that it promises to solve both the semantical and the soritical paradoxes at once. The argument to be developed below, however, raises a prima facie difficulty for the program. As we show in §6, some *predicates* themselves cannot 'detach' if they 'contract' (in a sense given in §6). If this is right, then v-curry shows that even languages devoid of any detachable conditional can exhibit curry-paradoxical features.

[13] A minor terminological point: one might prefer to more generally call it '*o*-Curry' for *operator*-Curry (or, strictly, *connective*-Curry) paradox; but any such $\odot$ exhibiting C1–C3 is near enough to being *conditional-ish* to warrant the tag 'c-Curry'.

particular, let γ be a sentence equivalent to $\gamma \odot \bot$. We may then reason as follows:

1.	$\gamma \leftrightarrow (\gamma \odot \bot)$	[T-biconditionals]
2.	$\mid \gamma$	[Assume, for '$\odot$ Proof' C4]
3.	$\mid \gamma \odot \bot$	[1, 2; MP]
4.	$\mid \bot$	[2, 3; C2]
5.	$\gamma \odot \bot$	[2–3; C4]
6.	γ	[1, 5; MP]
7.	$\bot$	[5, 6; C2]

Whatever the truth about the liar paradox (and its ilk), rcf theories all agree that robust contraction freedom is the key to c-Curry. We return to this diagnosis in §6. For now, we move on to consider what rcf theories say, or can say, about validity.

$\star\star$ *Parenthetical remark.* We should note that C4 is not necessary for the paradox; C3 will do the trick, but we appeal to C4 for uniformity of discussion. Without C4 the derivation, as per Restall (1993), runs thus:

1.	$\gamma \leftrightarrow (\gamma \odot \bot)$	[T-biconditionals]
2.	$\gamma \rightarrow (\gamma \odot \bot)$	[1; Simplification]
3.	$\gamma \odot (\gamma \odot \bot)$	[2; C1]
4.	$\gamma \odot \bot$	[3; C3]
5.	γ	[1, 4; MP]
6.	$\bot$	[4, 5; C2]

We should also note that if a connective $\odot$ satisfies C2 and C4, and the logic has Structural Contraction, the proof of $\odot$-contraction (i.e., C3), is straightforward but nonetheless instructive for present purposes:

1.	$\alpha \odot (\alpha \odot \beta)$	[Assumption]
2.	$\mid \alpha$	[Assume, for $\odot$ Proof, C4]
3.	$\mid \alpha \odot \beta$	[1, 2; C2]
4.	$\mid \beta$	[2, 3; C2]
5.	$\alpha \odot \beta$	[2–4; C4]

This derivation, too, presupposes the validity of Structural Contraction: α gets used twice in the subproof. (We return to this phenomenon below.) *End remark.* $\star\star$

3 Validity and truth-preservation

Truth theorists have worked to show how to get truth into our language—more accurately, a truth predicate that 'expresses truth', satisfying the T-schema. Similarly, such theorists have worked on the analogous problem of accommodating 'exemplification' in our language, where this satisfies the familiar exemplification schema (or, as it's sometimes called, *naïve comprehension*). The biggest challenge for all such tasks is c-Curry paradox;[14] and that challenge, on rcf approaches, is met by rejecting the existence of contracting connectives.

A natural next step, after accommodating *truth* and *exemplification* in our language, is to bring in a *validity predicate* to express validity.[15] In this section, we note a corollary of giving-up-Conditional-Proof approaches to C-curry that has been much-discussed recently.[16]

The notion of validity is often cashed out, at least intuitively, as 'necessary truth preservation'. At the very least, this is commonly thought to be a necessary condition of validity, where 'truth preservation' is a conditional claim with a conditional as consequent, namely, VTP (for 'validity truth preservation'):

(VTP) If an argument is valid, then if its premises are (all) true, its conclusion is true.

Where $\rightarrow$ is some conditional in the language supporting all instances of the T-Schema, and $Val(x, y)$ the validity predicate in and for the given language, VTP has the following form (for simplicity, we concentrate on single-premise arguments):

$$(\text{V0})\ Val(\ulcorner\alpha\urcorner,\ulcorner\beta\urcorner) \rightarrow (Tr(\ulcorner\alpha\urcorner) \rightarrow Tr(\ulcorner\beta\urcorner)).$$

As it turns out, rcf theorists—indeed, any theorists rejecting Conditional Proof but maintaining Structural Contraction—need to reject such a claim.

To see the problem, concentrate on the VTP principle. Omitting the truth predicate

[14] In the case of *exemplification*, one considers a semantical property $[x : x\epsilon x \rightarrow \bot]$ or, more generally, $[x : x\epsilon x \odot \bot]$ for a contracting connective $\odot$, the property exemplified by anything that exemplifies itself only if absurdity ensues, where the exemplification schema delivers $y\epsilon[x : x\epsilon x \odot \bot] \leftrightarrow (y\epsilon y \odot \bot)$, and then any of the c-curry derivations above go through with this replacing the T-biconditionals. See Beall (2008) for general discussion.

[15] This is not idle speculation. Many recent truth theorists have discussed the issue of adding a validity predicate. See e.g. Whittle (2004), Field (2008), Beall (2009) and Shapiro (2010a).

[16] See e.g. Priest (2006), Beall (2007b), Field (2008), Field (2009), Beall (2009) and Priest (2010).

for readability, we can think of VTP as V1,[17] namely,

$$\text{(V1) } Val(\ulcorner\alpha\urcorner, \ulcorner\beta\urcorner) \rightarrow (\alpha \rightarrow \beta)$$

The detachability of $\rightarrow$ (i.e., that MP is valid) amounts to the following claim (using the validity predicate):

$$\text{(V2) } Val(\ulcorner\alpha \wedge (\alpha \rightarrow \beta)\urcorner, \ulcorner\beta\urcorner)$$

But, then, by V1, V2 and MP we immediately get PMP, namely,

$$\text{(V3) } \alpha \wedge (\alpha \rightarrow \beta) \rightarrow \beta$$

Yet, as noted in §2, PMP is a notoriously easy recipe for c-Curry. For example, where γ is a Curry sentence equivalent to $\gamma \rightarrow \bot$, V3 implies triviality as follows:

1.	$\gamma \leftrightarrow (\gamma \rightarrow \bot)$	[T-biconditionals]
2.	$\gamma \wedge (\gamma \rightarrow \bot) \rightarrow \bot$	[Curry instance of V3, i.e. of PMP]
3.	$\gamma \wedge \gamma \rightarrow \bot$	[2; substitution of equivalents, viz. $\gamma \rightarrow \bot$ and γ]
4.	$\gamma \rightarrow \bot$	[3; substitution of equivalents, viz. $\gamma \wedge \gamma$ and γ]
5.	γ	[1, 4; MP for $\rightarrow$]
6.	$\bot$	[4, 5; MP for $\rightarrow$]

In rcf theories, truth preservation cannot be cashed out as V0.

Some rcf theorists, chiefly, Jc Beall and Hartry Field, have taken this to show that we must simply reject the claim that valid arguments are truth-preserving.[18] They have argued that this is not a defect of rcf theories, particularly when the conception of truth is one according to which *truth* is a mere transparent device—not explanatorily useful, and hence not used to explain or define validity. Other replies have also been advocated (e.g., Priest, 2006): VTP may not be expressed as V0, but, one suggestion goes, it may still be truly expressed in other ways—for example, in a non-detachable material fashion (see e.g. Beall, 2007b; Priest, 2010, pp. 134-5).

Our concern in this paper is not to dwell on the issue of truth-preservation and validity. Our aim is to highlight what we take to be a different issue in the

[17] In any *transparent truth theory* (Field, 2008; Beall, 2009) VTP is straightforwardly equivalent to V1, but we shall set aside exact details of the truth theories for present purposes.

[18] See Beall (2007b), Field (2008, Chapter 19), Field (2009, pp. 263-4) and Beall (2009, 34-41). Shapiro (2010a) gives a reply to the Beall–Field argument that bears on our current discussion. See §6 for discussion of Shapiro's program.

background: namely, expressing *validity* itself.

4 Varying the recipe: v-Curry

The lesson of c-Curry, we're supposing, is that Conditional Proof (CP) must be rejected. *If* this is right, what *v-curry* teaches—we now claim—is that the corresponding principle of *Validity Proof* (VP) is similarly problematic. Here, the basic idea is simply that if argument $\langle \alpha, \beta \rangle$ is in the validity relation, then $Val(\ulcorner \alpha \urcorner, \ulcorner \beta \urcorner)$ is true. Assuming that validity claims are appropriately 'necessary', so that validity claims are themselves valid if true, the point may be made in familiar notation using the turnstile as throughout, where this, as usual, picks out the validity relation for the target language:[19]

$$\text{(VP)} \;\; \text{If } \; \alpha \vdash \beta \; \text{ then } \; \vdash Val(\ulcorner \alpha \urcorner, \ulcorner \beta \urcorner).$$

In other words: if $\langle \alpha, \beta \rangle$ is in the validity relation, then saying as much—using the validity predicate—is true in a validity-strength fashion. (Compare VC from §2, and the corresponding *Conditional Proof*.)

In addition to VP, we also assume VD (for *Validity Detachment*, which, for a closer parallel with the c-version, one might call *v-MP*, though we stick with 'VD'):

$$\text{(VD)}\; \alpha, Val(\ulcorner \alpha \urcorner, \ulcorner \beta \urcorner) \vdash \beta.$$

In other words, even though $Val(x, y)$ is a predicate, it is clearly one for which it makes sense to attribute *detachability*. In particular, it is valid to infer (detach!) β from α together with the information that the argument $\langle \alpha, \beta \rangle$ is valid.

Putting VP and VD together yields what, by analogy with truth and exemplification, may be called the *V-schema*:

$$(\textit{V-Schema}) \quad \vdash Val(\ulcorner \alpha \urcorner, \ulcorner \beta \urcorner) \text{ iff } \alpha \vdash \beta.$$

What we now note is that VP and VD—or, simply, the V-schema—*along with the standard structural rules*, are the ingredients for *v-Curry* paradox. In particular, consider a sentence π equivalent to one saying that the argument $\langle \pi, \perp \rangle$ is valid—for example, in English, something like 'the argument from me to absurdity is valid', which, in

[19] We do not here pretend to be formulating this in a single, 'semantically self-sufficient' language, though this would seem not to be any more problematic than the case for truth. Here, one will simply have embedded $Val(x, y)$ claims.

T-biconditional form, may be represented formally thus:[20]

$$\pi \leftrightarrow Val(\ulcorner \pi \urcorner, \ulcorner \bot \urcorner)$$

We may then reason as follows:

1.	$\pi \leftrightarrow Val(\ulcorner \pi \urcorner, \ulcorner \bot \urcorner)$	[T-biconditionals]
2.	$\mid \pi$	[Assume, for VP]
3.	$\mid Val(\ulcorner \pi \urcorner, \ulcorner \bot \urcorner)$	[1, 2; MP]
4.	$\mid \bot$	[2, 3; VD]
5.	$Val(\ulcorner \pi \urcorner, \ulcorner \bot \urcorner)$	[2–4; VP]
6.	π	[1, 5; MP]
7.	$\bot$	[5, 6; VD]

What is plain, upon reviewing c-Curry in §1, is that *this* derivation has precisely the same structure as that for c-Curry. The difference between the two is that while c-Curry involves a conditional, v-Curry involves a predicate—notably, the *validity* predicate.

⋆⋆ *Parenthetical note.* We briefly digress to ask whether v-Curry is a *new* paradox. (One may skip to §5 to carry on the main discussion.) A number of works circle about the paradox, and some may have (independently) hit upon the paradox, which we are dubbing *v-Curry*. Our hope, in this paper, is to at least present the paradox as clearly as possible as one facet (or, if you like, flavor) of Curry's paradox.

Deutsch (2010, pp. 216-7) shows via what we would call a v-Curry-like argument that the *material conditional* may not be defined by means of a predicate $Impl(x, y)$ such that

$$Impl(\ulcorner \alpha \urcorner, \ulcorner \beta \urcorner) \leftrightarrow (\alpha \rightarrow \beta)$$

Deutsch shows, in other words, that you can't have a predicate expressing a connective for which a deduction-theorem link holds (e.g., the material conditional in a classical setting). This is correct, and important. We note, as we have above, that many current theories—certainly, the rcf ones—already lack a connective for which

[20] Given Gödel's Diagonal Lemma, our v-Curry sentences may be represented without using a truth predicate. To make things easier, however, we assume in the background a truth-predicate version, something such as 'I am true just if the argument from me to absurdity is valid'. Using our intuitive example from footnote 7, we can think of this phenomenon arising from a name c that, somehow or another, denotes $Val(\ulcorner Tr(c) \urcorner, \ulcorner \bot \urcorner)$, and so the T-sentences yield $Tr(c) \leftrightarrow Val(\ulcorner Tr(c) \urcorner, \ulcorner \bot \urcorner)$, to which we appeal below—though, as in the derivation above, we suppress the truth predicate for ease of reading.

a deductive-theorem link holds, and cannot have such a thing precisely for c-curry reasons. (We should also note that the material-conditional version of Curry's paradox is simply a disjunctive liar paradox, in effect, 'either I am untrue or everything is true'. Strictly speaking, this version of Curry's paradox is resolvable—and often taken to be resolved—by a theory of negation along paracomplete or paraconsistent lines. What makes Curry's paradox so difficult is that it arises even in negation-free languages. We hope that it is clear that the same applies to what we have dubbed *v-Curry*, a paradox that involves a *validity predicate*, arising even in languages devoid of negation.)

Similarly, Leitgeb (2007, p. 172) shows, via analogous reasoning, that a classical metatheory for the theory of truth presented by Field (2007) cannot contain a predicate $Impl(x, y)$ expressing Field's implication sign $\rightarrow$. This result does not assume a deduction-theorem link for target connectives; however, it focuses on the issue of whether candidate connectives are expressible via predicates (in a classical metalanguage) for them—and, so, not focused on *validity* itself, be it in Field's logic or other logics in the ballpark of our discussion.

Field (2008, p. 298 ff.) discusses a paradox turning on a sentence W that says of itself (only) that it is *inconsistent*, where *inconsistency*, in Field's discussion, is defined as *validly implying absurdity*. Taking $\perp$ to be an absurdity constant, Field's sentence W is essentially equivalent to what we are calling a *v-Curry* sentence: W is equivalent to $Val(\ulcorner W \urcorner, \ulcorner \perp \urcorner)$. Much of Field's discussion, while essentially related to (what we're calling) v-Curry paradox, is presented in a form much closer to standard liar-like reasoning than what we take to be the essential phenomenon: Curry's paradox. We briefly return to Field's discussion in §6.

Discussion of what we're calling *v-Curry* explicitly (and independently) shows up in papers by Whittle (2004, fn. 3) and Shapiro (2010a, fn. 29), who, while both concentrating on a *connective* version of Curry's paradox, explicitly point to what we're calling *v-Curry*. We briefly return to the Whittle and Shapiro programs in §6. We note that Shapiro's paper, while focusing on his program of 'deflating logical consequence', independently contains much of what we discuss here, and we see it as an important complement to this paper.

In his *Paradoxes from A to Z*, Michael Clark (2007, pp. 234-5) presents a similar, though perhaps not identical, paradox, which he attributes to Pseudo-Scotus. The paradoxical argument Clark considers is essentially the same as the one used in v-Curry, namely,

$$(\sigma)\ \frac{\text{This argument, } \sigma\text{, is valid.}}{\text{Therefore, } 1+1=3.}$$

But the *proof* he gives is different, as it relies on the assumption, rejected for other reasons by recent theorists (see §3), that valid arguments are necessarily truth-preserving:

> Suppose the premiss is true: then the argument is valid. Since the conclusion of a valid argument with a true premiss must be true, the conclusion of σ is true. So, necessarily, if the premiss is true, the conclusion is true, which means that the argument is valid. (Clark, 2007, p. 234)

We should also note that the name of Pseudo-Scotus is more often associated with what we may call a v-liar:[21]

$$(\tau)\ \frac{\top}{\text{This argument, } \tau\text{, is invalid.}}$$

As in Clark's version of the Pseudo-Scotus argument, a paradox is usually derived from τ on the assumption that valid arguments are necessarily truth-preserving: see, for example, Read (1979), Priest and Routley (1982) and Read (2001).

Recently, Field (2008, p. 305 ff.) considers a version of the v-liar

$$(\tau)\ \neg Val(\ulcorner \top \urcorner, \ulcorner \tau \urcorner)$$

but claims that it is not 'particularly compelling'. Priest (2010, p. 128) criticizes such a claim, pointing to a version of what we're calling v-Curry. He introduces a rule version of, respectively, VP and VD

$$\frac{\begin{array}{c}[\alpha]^n \\ \vdots \\ \beta\end{array}}{Val(\ulcorner \alpha \urcorner, \ulcorner \beta \urcorner)}\ \text{Val-I}, n \qquad \frac{Val(\ulcorner \alpha \urcorner, \ulcorner \beta \urcorner) \qquad \alpha}{\beta}\ \text{Val-E}$$

and argues that if every argument is either valid or invalid, as Field thinks, the v-liar gives us $\top \vdash \bot$.

While we agree with Priest that, in the paracomplete theories Field favours, a validity predicate may need to avoid Excluded Middle, we do not think that this gets to the heart of the matter. Just as c-Curry paradox is not automatically (if at all) resolved by restricting Excluded Middle, so too, we think, with what we've dubbed v-Curry paradox. *End note.*⋆⋆

[21] See e.g. Read (1979, p. 266, fn. 1).

5 Validity principles and revenge

Before briefly discussing a few avenues of reply, we want to emphasize the main point: namely, that, at least prima facie, resolving *c-Curry* and resolving *v-Curry* require the same solution. After all, they're two faces—or, as we've put it, two flavors—of the same paradox. The trouble, however, is that while breaking a deduction-theorem link between validity and a *conditional* is an option—and, indeed, a popular option—for avoiding c-Curry-driven absurdity, it is hard to apply with respect to the *validity* predicate itself. In particular, giving up VP seems not to be an option, at least if the validity predicate $Val(x,y)$ is to be the *validity* predicate—that is, expresses what follows from what, what stands in the validity relation (which we normally mark with the turnstile).

It may be thought that an alternative is to give up VD, thereby treating c-Curry and v-Curry in different but, in the end, closely related ways: the former teaches us that the *conditional* detaches (MP) but lacks a direct link with validity (no Conditional Proof); the latter teaches us that the validity predicate is the validity predicate, enjoying a direct link with *validity* (VP), but that it fails to detach (no VD). Some might think that there's at least some 'symmetry' in this asymmetric treatment of the two facets of Curry's paradox; however, we do not find the given 'symmetry of asymmetry' approach to be terribly plausible (or, pending further explanation, natural). While we admit no knockdown argument, it seems to us that VD is no less dispensable than VP if the validity-predicate is to express *validity*.

On this assumption, namely, that if both VP and VD are required in order for the validity-predicate to express validity, one might take our main argument to be a *revenge argument*. Consider the following recipe for revenge, as given in Beall (2007b, pp. 399-400):

(a) Find some semantic notion $\mathcal{X}$ that is (allegedly) in our natural language $\mathcal{L}$.

(b) Argue that $\mathcal{X}$ is not expressible in the truth theorist's formal language $\mathcal{L}_m$, the language that is supposed to formally explain why $\mathcal{L}$ isn't trivial, lest $\mathcal{L}_m$ be inconsistent or trivial.

(c) Conclude that $\mathcal{L}_m$ is explanatorily inadequate: it fails to explain how $\mathcal{L}$, with its semantic notion $\mathcal{X}$, enjoys consistency or non-triviality.

Armed with the foregoing recipe, we can easily get a revenge argument against rcf theories: just let $\mathcal{X}$ be validity and $\mathcal{L}_m$ be the rcf theorist's formal language. Revenge arguments, however, are never simple.[22] For example, the revenger insists that some notion $\mathcal{X}$, which is seemingly expressible in $\mathcal{L}$, *is* intelligible; however, the target truth theorists reject that $\mathcal{X}$ is intelligible—see, for example, Priest (2006) and Field (2008, p. 356) and Beall (2009, Ch. 3).[23] Whether the case of validity and, in particular, v-Curry-driven 'revenge' might be a special case—perhaps avoiding stalemate situations—is something that we leave open.

Our aim here, as stated above, is not to argue for one approach to Curry's paradox (in either flavor) over another. Our aim is only to highlight the two aspects of Curry's paradox and, in particular, their obvious structural similarity. On the other hand, once such similarity is noticed, a natural treatment of both versions emerges—a treatment at the 'structural' (indeed, *sub*structural) level. To this, and to other possible replies, we very briefly turn.

6 Avenues of reply

One avenue of reply for rcf theorists is to reject one of VP and VD, and concede that we don't have the resources to talk about validity. This line of response—one of 'silence', as it's sometimes called—is no more attractive in the case of validity than it is in the case of truth. On the face of it, we *do* talk about validity; and we should seek to account for this phenomenon, rather than deny the data, or deem it incoherent.

Perhaps less implausibly, one might go along a non-unified route waved at in §5. Instead of acknowledging a notion of validity that satisfies both VP and VD but about which we must remain silent, one might simply reject that there's a coherent notion of validity that satisfies both VP and VD (or, in short, the 'validity schema'). One approach in this direction might treat truth and validity as equally unstratified (or non-hierarchical) notions, but maintain that validity, unlike truth, fails to obey its apparently fundamental schema or corresponding rules (i.e., VP and VD). Along these lines, one might, as Field (2008, §20.4) suggests, treat v-Curry paradox as classical logicians treat Gödel-type phenomena: such sentences—including, now, what we call *v-Curry sentences*—are odd but ultimately *un*-paradoxical sentences. (E.g., on Field's suggestion, what we are calling *v*-Curry sentences might be seen,

[22]Shapiro (2010b) provides a useful discussion of the complexities involved in various sorts of revenge argument. See also Beall's introduction to Beall (2007a).

[23]The notion of being *just true* (false) for paraconsistent theorists and the notion of *being hyper-determinately true* (false) for paracomplete theorists *à la* Field—that is, determinately true (false) at all (transfinite) levels; see Field (2008, p. 326)—are cases in point. See also the cited Beall and Priest works.

from the standpoint of classical logic, as a Gödel sentence that asserts its own *dis*provability, a sentence treated as false but not disprovable in ZFC or a similar classical theory.) While this sort of response is coherent, as Field's discussion makes plain, it also carries an obvious awkwardness with respect to truth: it is difficult to see why *v*-Curry should undermine one of VP or VD (and, so, the corresponding 'validity schema') while *c*-Curry *fails* to undermine the corresponding T-schema (or, generally, 'truth rules'). If truth and validity are both understood as unstratified—more generally, non-hierarchical—notions, then both c-Curry and v-Curry prima facie demand a unified solution. The classical logician's situation with respect to Gödel phenomena, we think, is not sufficiently telling to overturn the prima facie demand of a unified solution to Curry's paradox.

Another avenue of reply immediately suggests itself: rcf theorists might treat truth and validity along very different lines with respect to their roles and 'nature'. Truth might be seen as a single, *unstratified* notion defined over the entire language (as rcf theorists in fact maintain), one that (as some rcf theorists maintain) has no important explanatory role, but rather only an expressive, logical one—along the lines suggested by disquotationalists or 'deflationists' generally. *Validity*, on the other hand, might be treated very differently: unlike truth, validity might be seen as an important explanatory notion that, as v-Curry might be taken to teach, is a *stratified* notion with many explanatory relations of validity at each 'level of explanation' (whatever such levels might come to). On this thought, *v*-Curry would be blocked for pretty much the same reasons that *c*-Curry is blocked if *truth* is taken to be a stratified notion: our paradoxical sentence $Val(\ulcorner\pi\urcorner,\ulcorner\bot\urcorner)$ would either become ungrammatical or would fail to express a proposition—one or the other, depending on the details of one's stratified approach.[24]

Bruno Whittle (2004), perhaps (independently) along the lines of Myhill (1975), advocates such a lesson, albeit in the much narrower context of *dialetheic* treatments of the semantic paradoxes. He writes:[25]

[24]Even on a broadly Tarskian approach, the stratification of *validity* isn't unavoidable. On the *contextualist* treatment of the liar paradox advocated by Charles Parsons (1974) and Michael Glanzberg (2004), *validity* would be a single and, in effect, non-stratified notion, but *valid propositions* would be ordered in an infinite—indeed, transfinite—hierarchy of contexts, each of which would come equipped with ever-larger sets of propositions available for expression. It is notable that on *transparency* conceptions of truth, such as Beall (2009) and Field (2008), the idea that *validity*—but not truth—can be treated in a stratified fashion may be thought to be acceptable (see e.g. Beall, 2009, p. 37), at least on the assumption that the validity predicate lacks the essential 'expressive role' of the truth predicate (or the *see-through device* or *disquotational device*). One aim of Shapiro (2010a) is to press against this sort of option for 'deflationists about truth', arguing instead for a uniformly 'deflationary' approach to both truth and validity.

[25]Strictly speaking, Whittle is wrong about 'the whole point' of such theories, at least if we include

> The whole point of [these] treatments [...] is their supposed avoidance of the sorts of hierarchies that are appealed to by more orthodox resolutions. However, [...] even if hierarchies are avoidable when talking about truth, they are not avoidable when talking about logical consequence. Thus, the supposed main advantage of these treatments would appear to be seriously undermined. (Whittle, 2004, p. 323)

We agree that, as we have put, *full Curry's paradox* affects more than one's treatment of connectives (in particular, conditionals); it also affects validity. But this means that, at least prima facie, notions such as truth and validity, both governed by very similar principles such as the T-Schema and the V-Schema, and both of which give rise to structurally identical paradoxes such as c-Curry and v-Curry, naturally call out for similar, *unified* treatments.

Unified treatments of the semantic paradoxes, however, do not abound. One option, to be sure, would be 'Tarskian', treating both validity and truth as equally hierarchical,[26] but Tarskian approaches face major and well-known difficulties, as Kripke (1975), Field (2008), and others have emphasized. Despite such difficulties, we note that, given the serious challenges of v-Curry paradox, unified Tarskian approaches may warrant renewed consideration. But we leave this to future debate, turning now to what is a more obviously unified approach suitable for target non-classical rcf theorists.

Instead of either treating truth and validity differently or 'going unified' along broadly Tarskian lines, one may extend the rcf lesson in the obvious fashion: just as c-Curry teaches us that our *connectives* don't contract, so too v-Curry teaches us that *validity* fails to contract. In other words, not only is contracting behaviour for our connectives (in particular, conditionals) to be rejected, but contraction at the *structural* level, namely, Structural Contraction,

$$\text{if } \Gamma, \alpha, \alpha \vdash \beta \text{ then } \Gamma, \alpha \vdash \beta$$

is to be rejected. For many non-classical logicians, this is prima facie the most natural

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rcf theories more generally, as we are doing here. Not only do rcf theories have the resources to generate Tarskian hierarchies for predicates other than truth, but some such hierarchies sometimes form an essential component of these theories. For instance, Field's theory of truth includes a hierarchy of determinacy operators of ever-increasing logical strength, each of which is definable in terms of Field's robustly contraction-free conditional (Field, 2007, 2008), and this feature is available, as Field points out, to rcf theories generally (because giving up contraction for one's connectives thereby provides a hierarchy of weaker and weaker connectives, from $\alpha \odot \beta$ to $\alpha \odot (\alpha \odot \beta)$, ..., and so on).

[26] For Tarskian treatments see Parsons (1974), Burge (1979), Williamson (1998) and Glanzberg (2004).

approach, given the similarity between c-Curry and v-Curry. This is particularly so for rcf theorists, where the idea—however radical—seems prima facie natural.

$\star\star$ *Parenthetical note.* We pause to note that, just as Whittle seems to advocate a hierarchical approach, Lionel Shapiro explicitly advocates a substructural approach (see Shapiro, 2010a) within a broader argument for the viability of what he calls *deflationism about logical consequence*. (He also notes that his arguments may be taken independently of his program of deflating consequence. We agree.) In short, Shapiro argues that just as deflationists about *truth* are committed to the T-rules,

$$\frac{\alpha}{\text{`}\alpha\text{' is true}}\ \text{T-I} \qquad \frac{\text{`}\alpha\text{' is true}}{\alpha}\ \text{T-E}$$

so too deflationists about *consequence* should be committed to the following *C-rules* (Shapiro, 2010a, p. 7):

$$\frac{\text{That } \alpha \text{ entails that } \beta}{\text{`}\alpha\text{' has `}\beta\text{' as a consequence}}\ \text{C-I} \qquad \frac{\text{`}\alpha\text{' has `}\beta\text{' as a consequence}}{\text{That } \alpha \text{ entails that } \beta}\ \text{C-E}$$

Shapiro then argues that if the deflationist's entailment connective satisfies

$$\text{`}\alpha\text{'} \vdash \text{`}\beta\text{' just if that } \alpha \text{ entails that } \beta,$$

as it should, then Curry-like reasoning—indeed, what we would call *v-Curry-like reasoning*—may lead deflationists about consequence to adopt a weakened version of MP: more precisely, one that effectively invalidates Structural Contraction (see Shapiro, 2010a, pp. 18-9).

We should also note that a similar substructural conclusion is anticipated by Priest and Routley (1982, p. 193ff), where a version of Curry's paradox is taken to preclude the 'suppression of innocent premises' within subproofs—effectively a rejection of Substructural Contraction within subproofs. [It is not at all obvious that later work of Priest (2006, e.g.) follows the lesson advocated in the given paper. See also other works by Priest cited here.] *End note.* $\star\star$

The substructural approach can be motivated—we now claim—by a generalization of the notion of robust contraction freedom (see §2). In very general terms, one extends the rejection of contracting *connectives* to a rejection of contracting *predicates*—including, in particular, the validity predicate.

What c-Curry shows, according to rcf theorists, is that our language is robustly contraction-free—devoid of any contracting *connective*. What *v-Curry* suggests, it seems to us, is that robust contraction freedom is not enough; it is at best enough only for c-Curry. What *v*-Curry calls for is *real* robust contraction freedom—in effect, robust contraction freedom *plus* freedom from any binary predicate H that satisfies the following three conditions:[27]

P1. $Val(\ulcorner\alpha\urcorner, \ulcorner\beta\urcorner) \vdash H(\ulcorner\alpha\urcorner, \ulcorner\beta\urcorner)$;

P2. $\alpha, H(\ulcorner\alpha\urcorner, \ulcorner\beta\urcorner) \vdash \beta$;

P3. $H(\ulcorner\alpha\urcorner, \ulcorner H(\ulcorner\alpha\urcorner, \ulcorner\beta\urcorner)\urcorner) \vdash H(\ulcorner\alpha\urcorner, \ulcorner\beta\urcorner)$.

To see this, consider the following version of v-Curry:

1.	$\pi \leftrightarrow Val(\ulcorner\pi\urcorner, \ulcorner\perp\urcorner)$	[T-biconditionals]
2.	$\mid \pi$	[Assumption, for VP]
3.	$\mid Val(\ulcorner\pi\urcorner, \ulcorner\perp\urcorner)$	[1, 2; MP for $\rightarrow$]
4.	$Val(\ulcorner\pi\urcorner, \ulcorner Val(\ulcorner\pi\urcorner, \ulcorner\perp\urcorner)\urcorner)$	[2-3; VP]
5.	$Val(\ulcorner\pi\urcorner, \ulcorner\perp\urcorner)$	[4; P3]
6.	π	[1, 5; MP]
7.	$\perp$	[5, 6; VD]

If VP and VD are beyond reproach, one cannot have any contracting *predicate* in the P1–P3 sense, and a fortiori $Val(x, y)$ itself cannot be as such. Given that, as is the case in the logics we're considering here, Identity holds (i.e., α is a consequence of itself), this means that one of P2 and P3 has to go. Robust contraction freedom is not enough. *Real* robust contraction freedom might be.

As far as validity is concerned, stratified or Tarskian theorists will arguably give up P3 for 'valid', or, if you like, *Predicate Contraction*, and substitute it with its stratified counterpart:[28]

$$Val_1(\ulcorner\alpha\urcorner, \ulcorner Val_0(\ulcorner\alpha\urcorner, \ulcorner\beta\urcorner)\urcorner) \vdash Val_0(\ulcorner\alpha\urcorner, \ulcorner\beta\urcorner)$$

Rcf theorists will likewise give up P3 for validity—that is, for the predicate 'valid'. Unlike stratified or Tarskian theorists, they will substitute it with nothing. This logical gap has some noteworthy consequences.

[27] If one thinks in terms of variable assignments and satisfaction, the following conditions can be given in a slightly eye-friendlier fashion thus:

P1. $Val(x, y) \vdash H(x, y)$;

P2. $Tr(x), H(x, y) \vdash Tr(y)$;

P3. $H(x, \ulcorner H(x, y)\urcorner) \vdash H(x, y)$.

[28] This is only one possible option, which would be certainly rejected by contextualist Tarskians such as Parsons and Glanzberg. It might be more natural in an 'indefinitely extensible' framework along the lines of Cook (2007).

Just as the proof of Contraction *tout court* requires Structural Contraction (see *infra*, fn. 12), so too does the proof of Predicate Contraction:

1.	$H(\ulcorner\alpha\urcorner, \ulcorner H(\ulcorner\alpha\urcorner, \ulcorner\beta\urcorner)\urcorner)$	[Assumption]
2.	$\mid \alpha$	[Assumption, for VP]
3.	$\mid H(\ulcorner\alpha\urcorner, \ulcorner\beta\urcorner)$	[1, 2; P2]
4.	$\mid \beta$	[2, 3; P2]
5.	$Val(\ulcorner\alpha\urcorner, \ulcorner\beta\urcorner)$	[2–4; VP]
6.	$H(\ulcorner\alpha\urcorner, \ulcorner\beta\urcorner)$	[5; P1]

Now let $H(x, y)$ be $Val(x, y)$. Then, P2 just is VD, and P1 just is Identity. Since, we're assuming, neither VP nor VD is to be rejected, only one option remains. Upon noticing that here, as in earlier derivations of contraction rules (e.g., C2, P2), an item (viz., α) gets used twice in the course of the subproof, the prima facie most natural diagnosis is that Structural Contraction has to go. While our aim is not to defend a substructural approach to v-Curry—or, indeed, to paradox in general—we briefly turn to a few remarks concerning such an approach.

To begin, it may be thought, not without reason, that dropping Structural Contraction defies belief. If we've assumed α, we are, it would seem, reasoning about a situation in which α is true. Call this situation s. Then, one might argue, surely it shouldn't matter how many times α is used while we reason about s, given that α is true in s. On this way of thinking, Structural Contraction would seem to be *essentially* built into ordinary reasoning.

We're sympathetic with this kind of concern. We note, though, that the worry only arises on certain—standard—conceptions of what validity is: for example, truth preservation in all possible situations, or worlds, or truth-preservation for all uniform substitutions of the non-logical vocabulary. If validity is conceived along such truth-preservation lines, then it is indeed very hard—though, admittedly, perhaps not impossible—to understand why Structural Contraction should not hold, except from the fact that *some of its uses* seemingly give rise to paradoxes.

Validity, however, may be conceived in ways other than along truth-preservation lines. Suppose, for example, that premises and assumptions are to be thought of as *resources* (Slaney, 1990; Paoli, 2002), as opposed to partial descriptions of worlds or situations or the like. Then it *does* matter whether β has been derived from, as it were, a *double-α resource*

$$\alpha, \alpha$$

rather than from the 'single-α resource' α alone. (Look again at the displayed Structural Contraction rule on page 17, conceiving of the α, α in the antecedent as one resource clearly different from the single α that follows Γ in the succedent.) In one case, α is a sufficient resource for β, but in the other, some more is required; and such additional resources may not be available if Structural Contraction isn't unrestrictedly valid.

On the other hand, it may be argued that conceiving of premises and assumptions as resources calls for a broadly *proof-theoretic* account of validity: premises and assumptions, one might say, are resources only if they are resources for *proofs*. And *this*, some might think, may be too hard to swallow. The point, on this thinking, is more general. What may be needed, if Structural Contraction is to be abandoned, is a new metaphysical account of validity—one for which the rejection of Structural Contraction is perfectly natural.[29]

A second potential difficulty arises: mathematics. John Slaney notes that structural contraction 'is deeply implicated in all sorts of paradoxical reasonings, from the sorites to Epimenides and from Russell's antinomy to Cantor's theorem' (Slaney, 1990, p. 78). Clearly, if a drastic revision of classical set theory is to be avoided, substructural logicians must find a way to discriminate between *paradoxes*, such as Curry's and the Liar, and *theorems*, such as Cantor's. It is not sufficient to point out that Cantor's theorem, and, for instance, classical set theory, can be recovered by assuming that certain uses of Structural Contraction are 'safe'. In addition to this, we must be told *why* these uses are safe, while other uses aren't, if substructural approaches to paradox are to be called *solutions* to the paradoxes in the first place. But we raise such issues not to solve them; we raise them only to indicate that, while *real robust contraction freedom* is a prima facie natural approach towards a unified solution to Curry's paradox (i.e., to both c-Curry and v-Curry), there are serious issues—both logical and philosophical—that demand attention on any such approach.

7 Closing remarks

Peter Geach, in discussing c-Curry, said this:

> If we want to retain the naïve view of truth, or the naïve view of classes [...], then we must modify the elementary rules of inference relating to 'if'. (Geach, 1955, p. 72)

[29]To our knowledge, little has been done along these lines, though Shapiro (2010a) offers an interesting direction for certain types of deflationary theories about truth and consequence.

Many contemporary theorists (Priest, 2006; Brady, 2006; Field, 2008; Beall, 2009, among others) agree with him, taking the c-Curry lesson to be that we need only adjust our 'operational rules' governing the behavior of our conditional and other such connectives.

What we hope to have illustrated is that there's more to Curry's paradox than c-Curry; there's v-Curry. The two versions of Curry's paradox are structurally equivalent, as we hope to have displayed. Hence, a unified treatment of Curry's paradox must treat the two paradoxes the same way. From a non-classical perspective on paradox—or, at least, from a robustly contraction-free perspective—the most natural unified solution takes the lesson to concern *structural rules*, and in particular involves a rejection of Structural Contraction. It remains to be seen, however, whether there's sufficient sense in the idea of *real* robust contraction freedom. We leave this for future debate; but we hope to have shown at least that the debate may well be worth having.

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