Truth, Necessity, and Abnormal Worlds

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INTRODUCTION

- Background: philosophy and truth theory (viz., BXTT) *Spandrels of Truth* (OUP, 2009), sometimes '*SoT*'.
- Current Aim: adding a plausible (broad) necessity operator to BXTT.
- There's quite a bit of background (e.g., everything in *SoT*!) that I'll quickly sketch!
- I will be skirting many issues along the way.

LIAR PARADOX

- Dialetheic solution: *gluts*.
- A glut is a truth with a true negation.
- Paraconsistency: $A, \neg A \nvdash B$.

Dialetheists can differ widely on the logic of truth and reach of 'true contradictions'. (I advocate a very, very conservative dialetheic position in *SoT*, though close to Priest's logical framework.)

For present purposes, I will speak (contrarary to fact) as if dialetheists all endorse the same basic theory/logic.

Problem: no suitable conditional!

Modus Ponens (MPP)—strictly, Material Modus Ponens (MMP)—is invalid.

- The material 'conditional' is a disguised disjunction: $\neg A \vee B$. (The disguise: $A \supset B$.)
- Suppose that $A \vee B$ is true iff A is true or B is true (or both).
- Suppose, now, that A and $\neg A$ are true.
- B itself needn't be true.

So, gluts present counterexamples to MMP. We need a suitable—detachable—conditional for our truth theory (e.g., T-biconditionals)!

SOLUTION: WORLDS AND PRIMITIVE CONDITIONAL

Introduce (or acknowledge) a collection $\mathcal{W}$ of 'worlds' and, in turn, a primitve conditional $\rightarrow$ which is *all-worlds-looking*.

$$w \models A \rightarrow B \text{ iff . } w' \models B \text{ if } w' \models A, \text{ for all } w' \in \mathcal{W}$$

In short: for *any* world w, our new conditonal $A \rightarrow B$ is true at w iff there's *no* world at which A is true but B not.

SOLUTION: WORLDS AND PRIMITIVE CONDITIONAL

- Our models contain a set of worlds $\mathcal{W}$.
- Disjunction, Conjunction get expected truth-at-a-world conditions (and falsity- too if need be).
- Negation gets truth-at-a-world conditions that allow for gluts (but, for current purposes, no gaps).
- Validity is as usual: absence of a world (in a model) that 'makes true' the premises but fails to 'make true' the conclusion.

Problem: PMP and Curry-driven triviality!

The chief problem with our new all-worlds-looking conditional (in an 'all-worlds access' universe) is that it validates *Pseudo Modus Ponens*.

$$\text{PMP.}\quad \vdash A \land (A \to B) \to B$$

Curry paradox combines with PMP and our T-biconditionals to generate triviality – absurdity.

FROM PMP TO CURRY TRIVIALITY

Let C be a Curry sentence that says $C \rightarrow \bot$ (e.g., 'If I am true, everything is true'), so that our T-biconditional – dropping $Tr(x)$ for simplicity – gives us $C \leftrightarrow (C \rightarrow \bot)$.

1. $C \leftrightarrow (C \rightarrow \bot)$ [T-biconditional]
2. $C \wedge (C \rightarrow \bot) \rightarrow \bot$ [PMP]
3. $C \wedge C \rightarrow \bot$ [2, substitution]
4. $C \rightarrow \bot$ [3, features of $\wedge$]
5. C [1,4 MPP]
6. $\bot$ [4,5 MPP]

So, we need to avoid PMP!

SOLUTION: ABNORMAL WORLDS AND JUMPY CONDITIONAL

Taking a page from Kripke: *introduce 'abnormal worlds' and allow our conditional to be 'jumpy' between types of worlds!*

- Our models acknowledge a non-empty set $\mathcal{N} \subseteq \mathcal{W}$ of 'normal worlds'.
- Define all (Boolean or standard first-order) connectives *uniformly* over all worlds.
- For our conditional: acknowledge 'jumpy' behavior, with $A \rightarrow B$ behaving one way at normal points and another way at abnormal points.

JUMPY CONDITIONAL: EXAMPLE

- For all *normal* worlds $w \in \mathcal{N}$:

$$w \models A \rightarrow B \text{ iff . } w' \models B \text{ if } w' \models A, \text{ for } \textbf{all } w' \in \mathcal{W}$$

- For all *abnormal* worlds $w \in \mathcal{W} \setminus \mathcal{N}$:

$$w \models A \rightarrow B \text{ iff} \ldots \text{[fill in favorite account (say, arbitrary)]}$$

* On these non-normal-worlds semantics, we *define validity only over (all)* **normal** *worlds of all models*.

JUMPY CONDITIONAL: EXAMPLE

With this setup, we keep MPP but, as wanted, lose PMP.

- MPP: validity is defined over normal worlds. For any normal world, $A \rightarrow B$ is true iff there's *no* $x \in \mathcal{W}$ at which A but not B is true. Hence, for any normal world w, if we've got $w \models A$ and $w \models A \rightarrow B$, we'll have $w \models B$.
- No PMP!! For *abnormal worlds*, we're treating the status of $A \rightarrow B$ in an arbitrary (or unrestricted ternary) fashion. So, just let $\mathcal{W} = \{x, y\}$ with $\mathcal{N} = \{x\}$, and let $y \models A$ and $y \models A \rightarrow B$ but $y \not\models B$. Then $x \not\models (A \wedge (A \rightarrow B)) \rightarrow B$ as there's a point y at which $A \wedge (A \rightarrow B)$ is true but B not.

(NB: the Routley–Meyer ternary relation gives a slightly less 'arbitrary' feel to things, but skip this topic here.)

Recap and Main Issue

- Liars motivate gluts.
- Gluts undermine MMP, and so push for a detachable conditional.
- Worlds and primitive all-worlds-looking conditional gives MPP.
- PMP and Curry paradox require *abnormal worlds* and 'jumpy' conditional.
- ...in the end, we have good (transparent) truth theories in this setup (and, thanks Aussie logicians, non-triviality proofs for many such theories).

Recap and Main Issue

The main issue concerns *necessity* (e.g., broad necessary truth).

- Now that we have worlds for our given truth theory, how are we to understand *necessary truth*?
- The task: add a plausible, broad-necessity Box to the picture (specifically, to the target theories).

One would think that this is straightforward, but there are some surprising obstacles. [NB: it *is* straightforward for the →-free theory! (See Priest RSL for basic ideas.) Our concern is the actual truth theory.] Before turning to the obstacles and an eventual solution, some desiderata should be noted.

Desiderata

These are minimal (though debatable) desiderata that are assumed (and that, pending argument, dialetheists ought not have to rule out).

- Necessitation: If $\vdash A$ then $\vdash \Box A$.
- Box Release (rule): $\Box A \vdash A$.
- Diamond Capture (rule): $A \vdash \Diamond A$ (where $\Diamond A$ is $\neg\Box\neg A$).
- K/Distribution (rule): $\Box(A \rightarrow B) \vdash \Box A \rightarrow \Box B$.
- S4/KK principle: $\Box A \vdash \Box\Box A$.
- S5 principles: $\Diamond A \vdash \Box\Diamond A$ or, in another form, $\Diamond\Box A \vdash \Box A$.

Of course, if some of these 'rule' forms can be strengthened to *axioms*, then all to the good, but we ask at least for the weaker (rule) forms.

Assumption: primitive Box

- Since we already have a non-triviality (analogously, consistency) result for the target theories, the best course would be having $\Box$ to be a *defined* connective.
- Unfortunately, I know of no viable source in the target theories in terms of which to define necessity (given the desiderata).
- (If there are ideas, please bring them up in discussion!)
- In what follows, we assume that $\Box$ is a primitive connective.

PROPOSAL 1: UNIFORM ALL-WORLDS (UAW)

Philosophers usually think of (broad) alethic necessity along 'all worlds' lines. This is a natural start. (The tag 'uniform' concerns no distinction between types of worlds—normal or abnormal.)

- We let $\mathcal{W}$ be our collection of worlds.
- We define our *uniform, all-worlds* (UAW) Box thus:

$$w \models \Box A \text{ iff } w' \models A \text{ for all } w' \in \mathcal{W}$$

UAW: PROBLEM

- Whatever other merits the UAW approach may have, we do not have Necessitation.
- In the target theories, we have $A \rightarrow A$ true at all *normal* worlds, and so $\vdash A \rightarrow A$.
- The problem: we do not have $A \rightarrow A$ true at *all* worlds, since there are *abnormal* worlds at which $A \rightarrow A$ is untrue.
- Hence: on the UAW approach, we have some A such that $\vdash A$ but $\nvdash \Box A$.

So, given the desiderata, we need to try something else.

PROPOSAL 2: UNIFORM ALL-*normal*-WORLDS

- Given that we have abnormal worlds around, it makes sense to restrict our Box only to such normal worlds.
- The current philosophy of abnormal worlds has them as 'worlds' that go beyond the logically possible.
- So, we want our alethic necessity operator to look only at *normal* worlds—only at worlds that are within logical limits (so to speak).

PROPOSAL 2: UANW

The current idea is to make explicit use of our (sub-) collection $\mathcal{N} \subseteq \mathcal{W}$ of worlds, namely the *normal* worlds.

- We define our *uniform, all*-**normal**-*worlds* (UANW) Box thus:

$$w \models \Box A \text{ iff } w' \models A \text{ for all } w' \in \mathcal{N}$$

In short: for *any* world w (of any sort), $\Box A$ is true at w iff A is true at all *normal* worlds (versus, as in UAW, *all* worlds).

What do we get from the UANW approach?

VIRTUES: UANW

This approach has expected virtues, some of which are:

- Necessitation holds. (If A is true at all *normal* worlds of all models, so is $\Box A$ on the UANW approach!)
- Many (if not all) of the other minimal desiderata are satisfied.

…but not all robustly virtuous things are purely virtuous. The UANW has a severe, knockdown defect: it engenders Curry-driven triviality!!

PROBLEM: UANW AND PMP!!

Whatever its other virtues, the UANW approach is untenable. Recall the trouble with PMP and Curry paradoxes.

- Define: let $A \Rightarrow B$ be $\Box(A \rightarrow B)$.
- Claim: $\vdash (A \wedge (A \Rightarrow B)) \Rightarrow B$.
- Proof: suppose $w \not\models_1 \Box(A \wedge \Box(A \rightarrow B) \rightarrow B)$ for some $w \in \mathcal{N}$, in which case there's some $x \in \mathcal{N}$ such that $x \not\models A \wedge \Box(A \rightarrow B) \rightarrow B$, and so there's some $y \in \mathcal{W}$ such that $y \models A$ and $y \models \Box(A \rightarrow B)$ but $y \not\models B$. As $y \models \Box(A \rightarrow B)$ we have $z \models A \rightarrow B$ for all $z \in \mathcal{N}$, and so *no world* (including y) makes A but not B true. Contradiction.

Problem: UANW, PMP, and Curry!

So, we get PMP for $\Rightarrow$. And this delivers Curry-driven triviality all over again.

- NB: T-biconditionals using $\rightarrow$ imply the $\Rightarrow$ version.
- So, just run the original Curry problem for our arrow all over again, replacing $\rightarrow$ with $\Rightarrow$.
- NB: what this shows is that, in Greg Restall's terminology, our theory is no longer robustly contraction-free: the UANW approach to $\Box$ creates a 'contracting conditional'.

What to do??!!

Diagnosis: UANW and PMP

- Curry paradox taught that our regular arrow had to be *jumpy*; it had to behave differently at abnormal worlds than at normal ones.
- On our UANW approach, it doesn't matter where in our universe of worlds we are (e.g., a normal or abnormal point); Box claims always look back to normal worlds.
- What's going on, then, is that our UANW approach to $\Box A$ forces A to be evaluated at normal points.
- And that's the problem: PMP is broken only by evaluating parts of it at abnormal points; and $\Box$-ed PMP doesn't get that choice.

PROPOSAL 3: JUMPY ALL-NORMAL-WORLDS

As with our conditional, so too with our Box: Curry paradox teaches that our Box is jumpy.

- For all *normal* worlds $w \in \mathcal{N}$:

$$w \models \Box A \text{ iff } w' \models A \text{ for all } w' \in \mathcal{N}$$

- For all *abnormal* worlds $w \in \mathcal{W} \setminus \mathcal{N}$:

$$w \models \Box A \text{ iff } w \models A$$

The 'normal' clause retains the UANW spirit, but is no longer uniform; it applies only to normal worlds. The 'abnormal' clause treats the Box as a truth operator at abnormal worlds.

Good News!

- We avoid $\Rightarrow$-PMP!! Countermodel: $\mathcal{W} = \{x, y\}$ with $y \in \mathcal{W} \setminus \mathcal{N}$. Let $y \models A$ and $y \models A \rightarrow B$ (say, on arbitrary evaluator or etc.) but $y \not\models B$. Then $y \models A \wedge \Box(A \rightarrow B)$ but, as above, $y \not\models B$.
- We get Necessitation: if $\vdash A$ then $\vdash \Box A$.
- K: $\Box(A \rightarrow B) \vdash \Box A \rightarrow \Box B$. (Not axiom!)
- S4: $\vdash \Box A \rightarrow \Box\Box A$

REMAINING ISSUES: S5 PRINCIPLES AND NEGATION

- Taking $\Diamond$ as $\neg\Box\neg$: the answer turns on negation.
  - Note: on the Star treatment, there's mixed news:

$$\Diamond A \nvdash \Box\Diamond A$$

    but

$$\Diamond\Box A \vdash \Box A$$

  - On an alternative (non-star) approach to negation, one may get both versions (though perhaps not axiom).
- Taking $\Diamond$ as primitive will probably give both options even in Star setting (but I'd prefer not to take it as primitive).

Remaining issues: non-triviality

I think that we get a relatively straightforward non-triviality proof for the resulting theory, but details remain open here.

Closing Remarks

- Dialetheists posit worlds in order to enjoy a proper conditional.
- Curry paradox teaches that abnormal worlds are also required for the conditional, and that the conditional needs to be 'jumpy'.
- In such a setting, it is natural to ask about broad necessity (or, dually, possibility): how does it work?
- I've shown that, at least in the target theories, Curry paradox reemerges when alethic necessity is added.
- I've also shown that the Curry-paradoxical lesson is the same: necessity, in the end, is abnormal—jumpy.

DISCUSSION IS OPEN!!!

...may discussion, unlike necessity, be uniform and normal.