Analetheism and dialetheism

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1. *Introduction*

Graham Priest (1987) argues that dialetheism alone avoids familiar revenge problems, and in that respect enjoys expressive virtues that rival theories lack.[1] In this paper we present a rival theory – analetheism – that seems to enjoy precisely the expressive virtues that dialetheism enjoys.[2] Analetheism, for us, is the thesis that some sentences lack truth-value, *coupled with the willingness to assert such sentences*. On a formal level, both theories rely on the same basic logic (viz. *LP*), differing only at restrictions placed on truth. On an informal level, the difference arises at interpreting *LP*'s third value: where the dialetheist sees truth-value gluts, the analetheist sees truth-value gaps.[3]

Our aim in this paper is not to argue for analetheism but rather to put it on the table with dialetheism; indeed, in terms of cost-benefit analysis, we do not see a clear 'winner' among the two positions. On one hand the analetheist, as we present her, evidently enjoys the same sort of expressive benefits enjoyed by the dialetheist (she avoids revenge problems); on the other, she also faces the same sort of (expressive) difficulties. Accordingly, we present the positions in bare parallel, leaving analetheism as a challenge for the dialetheist. Throughout, we attempt to say just enough to highlight the target, parallel, position.

In §2 we briefly review *LP*. In §3 we rehearse Priest's dialetheic take on the Liar and the dialetheic restrictions on truth.[4] §4 presents the analetheist's take on the Liar and her restrictions on truth. §5 provides some general observations about the two positions and their relationship to each other. In §6 we close by very briefly considering a different notion of truth (intersubstitutable truth) and its apparent effect on the two positions.

[1] Though we briefly rehearse a few basics, we assume familiarity with dialetheism and its expressive virtues. Sainsbury 1995 provides a nice introduction to dialetheism, and Priest 1987 presents an extensive discussion.

[2] The term 'analetheism' first occurs in Parsons 1990 for the thesis that paradoxical sentences lack truth-value; we use it slightly differently, with apologies for any confusion this may cause.

[3] That gluts and gaps are in some sense 'dual' has often been observed – for example, Dunn 2000, Parsons 1990, and Varzi 1999 – but, as far as we know, no analetheist position has yet been laid out. This paper begins the layout.

[4] For present purposes we will equate dialetheism with Priest's version of dialetheism (1987); that saves us from having to frequently write 'Priest's version of dialetheism'.

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2. *The logic of paradox*

LP is a three-valued logic: V (the set of values) is {1, 1/2, 0} and D, the set of designated values (assertable values, values to be preserved in valid inferences), is {1, 1/2}.[5] A valuation on the language is a function v from sentences into V. Valuation v is *admissible* iff it accords with the following diagrams:

¬	
1	0
1/2	1/2
0	1

&	1	1/2	0
1	1	1/2	0
1/2	1/2	1/2	0
0	0	0	0

v	1	1/2	0
1	1	1	1
1/2	1	1/2	1/2
0	1	1/2	0

The operator diagrams are precisely the $K3$ ('strong Kleene') diagrams; the difference is that 1/2 is designated in *LP*, undesignated in $K3$.

We will say that a valuation v *satisfies* a sentence A just if $v(A) \in D$, and that v satisfies a set of sentences X just in case v satisfies every element of X. Semantic consequence, $\models$, is then defined in the expected way:

$X \models A$ iff each admissible valuation that satisfies X also satisfies A.

LP's logical truths (consequences of the empty set) are precisely those of classical logic; however, *LP*'s consequence relation is significantly different. For example, *explosion* or EFQ fails in *LP*. (Let $v(A) = 1/2$ and $v(B) = 0$. Then v satisfies $\{A, \neg A\}$ but not B.)

For present purposes, both the analetheist and the dialetheist accept *LP* as the appropriate logic. Accordingly, both parties admit the (logical) possibility of designated (assertable) sentences of the form $A\&\neg A$. (Just let $v(A) = 1/2$.) The parties disagree on whether such designated sentences are true; each position imposes different conditions on truth. Thus, we turn to truth.

3. *Dialetheic truth*

The dialetheist asserts some false sentences – for example, Liar-like sentences. But that, according to the dialetheist, is OK; for assertability requires only that a sentence be *at least true*, and that is what (according to the dialetheist) the designated values represent. Given that some sentences of the form $A\&\neg A$ may be designated (and, hence, assertable), the dialetheist takes some sentences of the form $A\&\neg A$ to be (at least) true.

Dialetheism (Priest's version) takes the following principles to govern truth. Where $\rightarrow$ is a detachable but non-contraposable conditional (as per Priest 1987) and $\leftrightarrow$ its biconditional (formed in the usual way):

[5] For simplicity and space-considerations we present only the propositional fragment; the predicate extension is fairly straightforward and is covered in Priest 1987.

(DT1) $\mathrm{T}<A> \leftrightarrow A$
(DT2) $\neg\mathrm{T}<A> \rightarrow \mathrm{T}<\neg A>$

Rationales: (DT1), or 'the T-schema', is taken to be analytic, while (DT2) reflects the dialetheist's commitment to the non-existence of gaps.

One upshot: Let $v(A)$ be 1/2. Then $\mathrm{T}<A>$ may, in accordance with (DT1) and (DT2), have value 1 (informally, 'true only') or value 1/2 (informally, 'both true and false'). Priest gives up weak, or simple disquotational, truth.[6] If A is true and false then it is not false to say that A is true, and hence $\mathrm{T}<A>$ is true only – or so Priest argues. But the argument, Priest claims, is defeasible: in special cases (e.g. the Liar), $\mathrm{T}<A>$ can take value 1/2 when A does.

As is fairly well known, dialetheism enjoys significant virtues. Dialetheism answers the Liar paradox with no possibility of revenge. In turn, the dialetheic theory can achieve semantic closure – the language specified can contain its own truth predicate without expressive poverty, just as natural language seems to do.

Despite its extraordinary expressive virtues, dialetheism also 'enjoys' expressive difficulties – due not to expressive poverty but, perhaps, expressive abundance. The dialetheist, for example, wants to deny the existence of truth-value gaps. Let λ be '$\neg\mathrm{T}\lambda$' (a strengthened Liar). The dialetheist assigns 1/2 to '$\mathrm{T}\lambda$', and so, by *LP*'s rules for disjunction and negation, '$\neg(\mathrm{T}\lambda \vee \neg\mathrm{T}\lambda)$' is designated and thus is assertable (because at least true). The intended interpretation of '$\mathrm{T}\lambda$' (that λ is true) renders '$\neg(\mathrm{T}\lambda \vee \neg\mathrm{T}\lambda)$' as the claim that λ is neither true nor not true, certainly not something the dialetheist wants to say given her aversion to gaps. The dialetheist says that λ is a special case, and it is; but it remains an awkward commitment.

4. Analetheic truth

The analetheist asserts some gappy sentences – for example, Liar-like sentences. But that, according to the analetheist, is OK; for assertability requires only that a sentence be *at least not false*, and that is what (according to the analetheist) the designated values represent. Given that some sentences of the form $A\&\neg A$ may be designated (and, hence, assertable), the analetheist takes some sentences of the form $A\&\neg A$ to be (at least) not false. The value 1/2, informally read as *neither true nor false*, is assertable because unfalse, according to analetheism.

The analetheist takes the following principles to govern truth, where $\rightarrow$ and $\leftrightarrow$ are per §3:

[6] We briefly return to the idea of simple disquotation in §6.

(AT1) $\neg \mathrm{T}<A> \leftrightarrow \neg A$
(AT2) $\mathrm{T}<\neg A> \rightarrow \neg \mathrm{T}<A>$

Rationales: If, as the analetheist believes, some untrue sentences are assertable (because unfalse), then the T-schema (DT1) ought to fail; however, certainly *something* like the T-schema is correct (possibly even analytic), and the analetheist is happy to accept its contraposed form (AT1), which the dialetheist rejects. (AT2) simply reflects the analetheist's rejection of the existence of truth-value gluts.

One upshot: Let $v(A)$ be 1/2. $\mathrm{T}<A>$ may, in accordance with (AT1) and (AT2), have value 0 (informally, 'false') or value 1/2 (informally, 'neither true nor false'). As with Priest, the analetheist gives up weak, or simple disquotational, truth. If A is neither true nor false, then it's false to say that A is true, and hence $\mathrm{T}<A>$ is false – or so the analetheist holds, paralleling her dialetheic counterpart.[7] Again, the argument is defeasible: $\mathrm{T}<A>$ can, in special cases (e.g. the Liar), take value 1/2 when A does.

Analetheism shares in dialetheism's virtues: since the analetheist is willing to assert some sentences of the form $A \& \neg A$ (because they are at least not false), no revenge problem threatens, and thus semantic closure is achieved as fully as by the dialetheist. The virtues of dialetheism are evidently enjoyed by analetheism.

Unsurprisingly, there is trouble precisely parallel to the dialetheist's. The analetheist, for example, wants to deny the existence of truth-value gluts. Let λ again be a typical strengthened liar sentence. The analetheist assigns 1/2 to '$\mathrm{T}\lambda$', and so, by *LP*'s rules for disjunction and negation, '($\mathrm{T}\lambda$ & $\neg\mathrm{T}\lambda$)' is designated and thus is assertable (because at least not false). The intended interpretation of '$\mathrm{T}\lambda$' (that λ is true) renders '($\mathrm{T}\lambda$ & $\neg\mathrm{T}\lambda$)' as the claim that λ is both true and not, certainly not something the analetheist wants to say given her aversion to gluts. The analetheist, like her dialetheic counterpart, says that λ is a special case, and it is; but it remains an awkward commitment.

5. *General observations*

As far as we can see, the analetheist achieves whatever expressive virtues that the dialetheist achieves; and she also partakes of the same sort of expressive vices as the dialetheist. What could tip the scales in favour of one position over the other? We do not know.

Each position, however stated, is initially a bit awkward: both parties assert some sentences of the form $A \& \neg A$. The dialetheist concludes from

[7] The analetheist thus accepts Dummett's famous argument from gaps against the T-schema (1978). (The dialetheist, of course, accepts the 'flip-side' of the argument against (AT2).)

the assertability of some such sentences that they are true (while dogmatically accepting that only truths may be asserted); the analetheist concludes that some untruths are assertable (while dogmatically accepting that no sentence of the form $A\&\neg A$ is true). Each position runs counter to one traditional dogma while accepting another, but neither position seems to us to outstrip the other in either costs or benefits. We thus leave the matter for further debate, leaving analetheism as a challenge to the dialetheist.[8]

6. *Closing remarks*

We close with one final observation. The intersubstitutability of *A* and T<*A*> is a common desideratum for a truth predicate. Both dialetheism and analetheism, at least as here sketched, give up such intersubstitutability. Suppose, however, that we require it. What happens to our two theories?

They become one. The resulting theory – *LP* with intersubstitutable truth – endorses each of (AT1), (AT2), (DT1), and (DT2). As a result, for any sentence *A* that receives the value 1/2, the current theory licenses both the assertion *that A is true and false* and the assertion *that A is neither true nor false*. The dialetheist and analetheist are no longer distinguishable on a simple disquotational (intersubstitutable) notion of truth, at least given *LP*. Accordingly, an *LP*-based 'disquotational dialetheist' might just as well be called an *LP*-based 'disquotational analetheist' – or perhaps some hybrid name.[9] But we leave that issue open.[10]

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[8] A further note: it may seem that analetheism is not a proper challenge to dialetheism, but rather a variety of dialetheism. If all there is to dialetheism is the willingness to assert some sentences of the form $A\&\neg A$, then that is right – analetheism is a form of dialetheism. But we think that that misses the point; the analetheist accepts, and the dialetheist rejects, that no sentence of the form $A\&\neg A$ is true. Both parties assert $A\&\neg A$ (for some *A*) but they differ significantly on whether such an assertion is (at least) true.

[9] Beall (forthcoming) advocates just such an *LP*-based 'dialetheic deflationism'. Field (forthcoming) also observes the awkwardness of combining intersubstitutable truth with *LP*-based dialetheism.

[10] We are grateful to Tom Bontly, Otavio Bueno, Mark Colyvan, Daniel Nolan, Graham Priest, Greg Restall and an anonymous referee for comments on an earlier draft, and to Hartry Field and Stewart Shapiro for discussion of related issues.

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